## Contents

		edgeme i Semi	ents nara, Stefano Lanzoni, Nicoletta Tambroni	7
		duction		9
1.	1.1		onale behind the present Monograph	9
	1.2	Plan	a of the Monograph	11
2.	Incon	npressi	ble hydrodynamics in straight channels	15
	2.1	The	motion of viscous incompressible fluids	15
		2.1.1	Stress	15
		2.1.2	Constitutive law	16
		2.1.3	Governing equations	16
		2.1.4	Boundary and initial conditions	18
	2.2	Turb	pulent flows	19
		2.2.1	General features of turbulence	19
		2.2.2	The energy cascade	21
		2.2.3	Investigating turbulent flows	22
		2.2.4	The Reynolds averaged formulation for turbulent flows	26
		2.2.5	The RANS equations	26
		2.2.6	Boundary conditions for the RANS equations	27
		2.2.7	Transport equation for the mean vorticity	29
		2.2.8	The closure problem	29
		2.2.9	Wall-bounded flows: plane uniform turbulent flow	32
	2.3	Turb	oulent flow in straight cylindrical open channels	37
		2.3.1	Geometrical preliminaries	37
		2.3.2	Formulation	38
		2.3.3	Uniform steady (normal) flow.	39
		2.3.4	The case of infinitely wide rectangular channels.	41
		2.3.5	The case of channels with finite width and arbitrary shape	43
		2.3.6	Turbulence driven secondary flows in channels	46
	2.4	Dept	th averaged formulation: the shallow water approximation	48

FUP Best Practice in Scholarly Publishing (DOI 10.36253/fup\_best\_practice)

Giovanni Seminara, Stefano Lanzoni, Nicoletta Tambroni, *Theoretical Morphodynamics: Straight Channels*, © 2023 Author(s), CC BY 4.0, published by Firenze University Press, ISBN 979-12-215-0213-8, DOI 10.36253/979-12-215-0213-8

		2.4.1	Pressure is hydrostatically distributed	49
		2.4.2	Derivation of the shallow water equations	50
		2.4.3	Closures	53
		2.4.4	Boundary conditions	54
	2.5	One-	dimensional formulation: de Saint-Venant equation	54
		2.5.1	Derivation of the 1D governing equations	55
		2.5.2	Closures	57
		2.5.3	Boundary conditions	57
3.	The s 3.1		nt transport capacity of open channel flows motion of an isolated heavy particle in viscous fluids	59 59
		3.1.1	Small particles: The equation of Maxey-Riley-Gatignol	60
		3.1.2	The particle response time and the Stokes number	62
		3.1.3	Larger particles	63
		3.1.4	An important application: Settling of a sphere in a fluid at rest at moderate-large Reynolds numbers	64
	3.2	The	modes of sediment motion	65
		3.2.1	Transport mode versus sheet/debris mode	65
		3.2.2	Transport mode: bedload versus suspended load	67
		3.2.3	The modern view: coherent turbulent structures and their role in particle entrainment	67
	3.3	Bedle	oad transport	69
		3.3.1	Threshold conditions for particle entrainment as bedload	69
		3.3.2	Bedload transport capacity of homogeneous sediments in uniform open channel flow	72
		3.3.3	The recent contribution of computational fluid dynamics (CFD) to modeling sediment transport in open channels.	78
	3.4	Thre	shold conditions for particle entrainment in suspension	82
		3.4.1	The entrainment mechanism	82
		3.4.2	Incipient particle entrainment	83
	3.5	Mode	eling the dynamics of suspensions	85
		3.5.1	Modeling techniques	85
		3.5.2	The Eulerian approach: governing equations of sediment mixtures	87
		3.5.3	Governing equations of the dynamics of suspensions of uniform sediments in the dilute approximation	90
		3.5.4	Equilibrium transport of dilute turbulent suspensions of small particles in open channel flows: the equilibrium Eulerian approach	93
4.	Sedin	nent tra	ansport in natural channels	101
	4.1	The	evolution equation of the bed interface	101

4.2	Bedlo	oad transport of homogeneous sediments in non-uniform flows	103
	4.2.1	Spatial non uniformities	103
	4.2.2	Temporal non uniformities	103
4.3	Non	uniform dilute suspensions of homogeneous sediments	103
	4.3.1	The 3D Reynolds averaged advection-diffusion equation	103
	4.3.2	Depth averaged model of transport in suspension	105
	4.3.3	Analytical relationship for the depth integrated suspended sediment flux per unit width appropriate to slowly varying flows	111
4.4	Bedlo	bad transport of homogeneous sediments on sloping beds	113
	4.4.1	Threshold conditions for the inception of sediment motion on sloping beds	113
	4.4.2	Bedload transport on sloping beds: dimensional arguments	118
	4.4.3	Bedload transport on sloping beds: Theoretical and experimental estimates	119
4.5	Flow	resistance in natural channels	123
	4.5.1	Small scale fluvial bedforms	123
	4.5.2	Bedform regime	124
	4.5.3	Bedform characteristics at equilibrium	126
	4.5.4	Bedforms and flow resistance	128
	4.5.5	Bedforms and sediment transport	132
	4.5.6	Small scale fluvial bank-forms	132
4.6	Sumr	nary of the mathematical formulation for homogeneous sediments	135
4.7	Exter	nsions to heterogeneous sediments	137
	4.7.1	Threshold for particle entrainment of heterogeneous sediments: Hiding	137
	4.7.2	Evaluation of grain size specific bed load transport capacity	139
	4.7.3	Grain size specific formulation of Exner equation: Hirano's approach	142
	4.7.4	Grain size specific formulation of Exner equation: The missing role of vertical sorting	144
4.8	Appe	ndix: Bedload transport of homogeneous sediments on finite slopes	146
The b 5.1		ate: Straight channels at equilibrium pasic <i>equilibrium</i> profile of straight channels	$153 \\ 153$
0.1	5.1.1	The notion of morphodynamic equilibrium	153
	5.1.2	Mathematical formulation of 1D morphodynamic equilibrium	156
	5.1.3	The case of cylindrical channels	156
	5.1.4	The case of cylindrical channels: rectangular channels with variable	100
	J.1.1	width	159
	5.1.5	The case of non cylindrical channels: natural rivers	165
	5.1.6	Additional effects	168
5.2	The h	pasic equilibrium cross-section of straight channels	168

5.

## CONTENTS

		5.2.1	The early field observations	168
		5.2.2	The equilibrium shape of channels unable to transport sediments	169
		5.2.3	The equilibrium shape of channels able to transport sediments: solution of the stable channel paradox for gravel rivers	172
		5.2.4	The equilibrium shape of channels able to transport sediments: sand rivers	s176
6.	Free a	and for	ced bars in straight channels	181
	6.1	Free	and forced bars in the laboratory	182
	6.2	Bars:	The simplest theoretical framework.	184
	6.3	The i	formation of free bars: Linear stability analysis	189
		6.3.1	Bar formation under bedload dominated conditions	189
		6.3.2	Free bar formation: the effect of suspended load	199
		6.3.3	Convective versus absolute instability	200
		6.3.4	A conceptual digression: is 2D morphodynamics a well-posed problem?	201
	6.4	Force	ed bars in straight channels	201
		6.4.1	Excitation of non migrating spatial modes in straight rectangular channels	201
		6.4.2	Bars forced by channel width variations	205
	6.5		development of free and forced bars of finite amplitude: weakly near theories	212
		6.5.1	Weakly nonlinear free bars	212
		6.5.2	Weakly nonlinear forced bars	219
	6.6	Fully	nonlinear free and forced bars	220
		6.6.1	Numerical simulations of finite amplitude free bars	220
		6.6.2	Numerical simulations of finite amplitude forced bars	227
		6.6.3	Interaction between free and forced bars in straight cohesionless channels	228
	6.7	Bars	in the field: additional complexities of bar morphodynamics	230
		6.7.1	The effect of flow unsteadiness	230
		6.7.2	Recent field observations of alternate bar migration	236
		6.7.3	The role of sediment heterogeneity: sorting patterns	240
		6.7.4	The role of insufficient sediment supply	248
		6.7.5	Further effects that influence the development of alternate bars in the field	l 249
7.	Intro 7.1		to the morphodynamics of mixed alluvial-bedrock straight channels channel limited versus transport limited	251 251
	7.2	Mech	anisms of bedrock erosion	252
	7.3	Morp	hology of bedrock channels	253
		7.3.1	Large scale morphological features of bedrock channels	254
		7.3.2	Small scale morphological features of bedrock channels	254
	7.4	Early	morphological models of the reach-basin scale	257

4

## CONTENTS

	7.5	Mech	nanistic models of bedrock erosion	259
		7.5.1	The saltation-abrasion model	260
		7.5.2	Developments of Sklar-Dietrich model	263
		7.5.3	Extension of the saltation-abrasion model to total load	268
	7.6	Mech	nanics of bedrock channel incision	273
		7.6.1	Insight on the bedrock incision process from laboratory observations	273
		7.6.2	1D incision models	278
		7.6.3	Stability of the longitudinal bed profile and autogenic development of knickpoints	282
		7.6.4	The interplay of incision and alluviation in shaping the cross section of bedrock channels	283
	7.7		ards a theoretical framework for the morphodynamics of mixed cock-alluvial channels	287
		7.7.1	Particle entrainment in mixed bedrock-alluvial channels	287
		7.7.2	Modeling bedload transport in mixed bedrock-alluvial channels	289
		7.7.3	Bed evolution equations for mixed bedrock-alluvial rivers	290
		7.7.4	Consistency of the theoretical framework: Free bars in mixed bedrock-alluvial channels	291
		7.7.5	The effect of macro-roughness	294
8.	Epilo	gue		297
	8.1	Achi	evements and limits of recent developments	297
	8.2	Fina	l warning	301
9.	Math	ematic	al Appendix	303
	9.1	Asyn	nptotic expansions	303
		9.1.1	A simple example of asymptotic representation	303
		9.1.2	Asymptotic expansions	305
		9.1.3	The asymptotic forms of some integrals: method of steepest descents	306
	9.2	Elen	nents of perturbation methods	308
		9.2.1	Weakly damped linear oscillator: exact solution	308
		9.2.2	Perturbation solution based on a straightforward expansion	309
		9.2.3	Dealing with non-uniformities: The method of multiple scales	311
	9.3	An i	ntroduction to stability theory	315
		9.3.1	The general notion of stability	315
		9.3.2	The toy model	317
		9.3.3	The base state	318
		9.3.4	Linear stability analysis: Temporal normal modes	318
		9.3.5	The initial value problem for linear perturbations and the convective-absolute nature of the instability	e 321

## CONTENTS

	9.3.6 Weakly nonlinear stability analysis	326
10.	Bibliography	337
11.	Notations	363