Foreword

1.1. Motivations and content of the volume

As the title suggests, this book collects notes that were prepared for a university course I taught in the Spring of 2018, with a slightly different title from the one I chose for this book, *Probability and Rational Choice*, and delivered to an audience of students enrolled in the Master course on *Logic, Philosophy and History of Science* (*Laurea Magistrale in Logica, Filosofia e Storia della Scienza*), held at the *Dipartimento di Lettere e Filosofia* of the University of Florence. The goal of the course was to introduce students to some basic concepts from the area of research generally known as decision theory. Due to the vastity of it, it was necessary to sharpen the aim of the course by carefully selecting some significant topics to expose the students to. I have made my choice in this respect as is usual the case for situations like this one, namely by taking into account factors, like expertise, taste, opportunity, etc., that could make the selection optimal for the course, and for the Master programme objectives, or, I should better say, for my understanding of them.

The Master programme this course was taught for the sake of, is an interdisciplinary curriculum which aims at teaching students coming from different bachelor studies, either settled within the Humanities, or linked to scientific disciplines. Course topics vary, but are in general somehow related with actual researches centered on Logic or on the Philosophy and the History of Science. This is a unique thing among the Master programmes available at the Academic level in Italy. A substantial part of this objective is achieved by introducing students to the use of formal methods, in agreement with a tradition that has made the philosophical studies in Florence renowned in, and outside Italy. The course I have taught, and this book which, as I said, comes out from that teaching experience, was intended to serve this purpose.

Riccardo Bruni, *Course notes on finite games and rational choice*, ISBN 978-88-6453-832-7 (print)
ISBN 978-88-6453-831-0 (online), CC BY 4.0, 2018 Firenze University Press

The concept which I decided to put under the spotlight is the concept of « rational choice». I also decided that the methodological tool that could be mostly useful to deal with it was the theory of games. This is because this latter area of research has rapidly become a territory where different scholars, mathematicians, economists, philosophers, meet for discussing topics from multiple points of view. Therefore, I thought that sticking to games could best reflect the educational aim the course was supposed to have. In particular, since I was trying to minimize prerequisites needed to attend the course (see below), I decided to limit myself to the treatment of *finite* games.

Regarding the selection of topics in that direction of work, I doubt that the expert reader would consider it an original one: it is rather exactly what one may expect an introductory volume to decision theory centered around the tool of finite games to contain.

Chapter 1 is my attempt to motivate the investigation on choice – on rational choice to be more precise –, from the game-theoretic angle I will adopt and propose in the subsequent parts of the volume.

With chapter 2 I start to elaborate the main dichotomy upon which the book is built, namely as based on distinguishing the analysis of games where the order in which players play is not taken into account, which yields the theory of games *in normal form* as they are commonly referred to, from the approach to games where that feature is present, that is to games as seen in *extensive form*. Beside laying down some basic terminology, this part of the book aims at making clear the implications in 'geometrical' terms involved in taking the one route, and in taking the other instead.

Having making clear the two areas of knowledge we will be interested in, I present in chapter 3 the theory of finite games in normal form (or, I should rather say, the part of it I decided to select for the volume's sake). Once again, I must stress that nothing unexpected happens here, topic-wise: after discussing the notion of Nash equilibrium as a natural generalization of what could be seen as representing a 'solution' to a game in normal form (that is, a solution of the choice problem that can be formulated for it, namely: What action should each player choose?), I go on questioning the natural character of this concept by discussing situations, in the form of games, in which by sticking to equilibria one seems to deviate from paths that rational agents might be expected to prefer. Refinements of the original notion of equilibrium that stem from the relevant critiques to it, at least some of those that have been proposed in the literature, are presented along the way. If this, as I said, was more or less expected, what is unexpected maybe, hence counts as an original contribution that this monograph put forth in this respect, is the approach by means of which the basic properties of equilibria and their refined versions are attained at, and proved to hold. This is done by making use

of formal methods which are proper to the logical investigation and are based on the setting-up of a formal language to speak of actions, utilities associated to them and their comparison. A definition of «rational strategy» is then introduced on this basis, and a study of it is pursued by means which are again standard in areas of logical research where circular concepts are relevant (like, for instance, the theory of formal truth). The whole story is told with some more bibliographical details in section 3.12.

Chapter 4, the last one of the book, contains the treatment of games in extensive form. Here, the diagrammatic form of trees by means of which games of this type are sometimes accounted for in the literature, is abandoned in favour of the mathematical model of games as sequences of natural numbers. After providing the reader with the concepts needed to familiarize with the approach, as well as a discussion about alternatives to the model that we call «canonical» and which come out from reflecting upon the common experience with games in real life, it follows a presentation of (what I chose to be) the main result on games in this form, namely the theorem of determinacy. The proof idea bears similarities with the aforementioned methodology used to deal with games in normal form. In particular, a formal language is introduced in order to speak of games in the chosen form and show that determinacy turns out as a consequence of some validity properties of formulas expressing that a strategy for winning every match of the game under scrutiny exists for one of the two players.

Presuming that by what I have said so far I succeded in providing the reader with a credible answer to the question «What do I get back if I decide to pay the price needed to go through the content of this book?», the other information required to make the decision, and eventually paying the actual price needed to buy the volume, is «What does the reading of this book presume I know already?». Being this latter question as important as the former, let us spend a few words about it as we did for the sake of answering the other one.

1.2. Notation and prerequisites

Having said that the book is supposed to be an introduction to the topics it treats, the best answer to the question on prerequisites is: nothing, no previous knowledge is presupposed here. To say that would turn out to be a lie. It would have to be regarded as a small lie probably, but still a lie in the end. Since to set up a relationship on lies is never a good idea, I will try not to lie to the reader, with whom I am suppose to engage a relationship henceforth. So, this book does indeed require some pre-existing knowledge. I have tried to avoid any assumptions about the reader knowing something about finite games and being acquainted with

the notation at use in that area of study. Since the choice of sticking to games as a tool for approaching the theory of rational decision was a deliberate choice of mine, and certainly a different route could have been chosen instead, I did not want to make any presupposition regarding the reader's expertise in the area. So, the volume aims at being a genuine introduction to the theory of finite games for beginners.

As the approach that is fostered here to game-theoretic topics is partially novel, prerequisites have to be evaluated with respect to that too. Both the treatement of finite games in normal form, for what concerns the theory of equilibria and their refinements, as well as the theory of games in extensive forms as sequences, in particular the proof of the theorem of determinacy from section 4.8, are based on the use of formal methods that are proper to the logical investigation.

Although I have tried both to motivate the methodological choice and to provide the reader with the necessary information step by step, I could not avoid assuming familiarity with the way in which formal languages are dealt with notationally in the first place, and how the basic notions involved in the costruction of them, such as the concepts of terms and formulas, are commonly introduced. So, the reader who has already been exposed to the standard notation concerning predicates application, symbols chosen for the main logical operations and the like, will experience no problem in going through the sections from chapter 3 and chapter 4 where they are at use. In addition, familiarity with inductive definitions of terms and formulas, as well as with proofs by induction, might be presupposed by some of the passages the reader will be required to go through. Parts of chapter 3 (in particular, section 3.5) and chapter 4 may also require familiarity with basic mathematical notions which are only partially accounted for here. Some very basic knowledge of set theory, both notationally and conceptually speaking, and a very basic acquaintance with the mathematical notion of function could ease the reading of those parts of the volume.

That is all the reader should be warned of, I think. I have decided to publish these notes both to serve an 'internal' purpose (that is, to provide future students of other courses I may deliver on the topic with a manual they could use as a companion to actual class notes), as well as an 'outer' one, namely to share them with scholars, make them available to students and, why not, teachers of other courses that may be in search of a compact manual on these topics. I hope that anyone who will happen to stumble on this book will find it a useful tool for reading, researching, studying and teaching.

Before launching ourselves into the actual reading of the volume, I would like to save a few words for acknowledgements. As a matter of fact, there is quite a number of people I should express my gratitude to as far as the preparation of this material for publication is concerned. First of all,

to the family at large at home. Secondly, to my collegues of the research group in Logic and Philosophy of Science, and to Andrea Cantini in particular, for several discussions concerning topics treated here that finally brought me to develop the interest needed to conceive the writing of this book. To the students, who attended my course in the Spring semester of 2018 for being the first ones to test these notes and for reporting me some typos they originally contained. Finally, to the Dipartimento di Lettere e Filosofia as a whole, that granted me a generous support for making this publication possible. It is intended that I remain the only responsible for mistakes and inaccuracies that might still be found here.

Firenze, 17 September 2018