The Development of Descartes' Idea of Representation by Correspondence

Hanoch Ben-Yami

Abstract: Descartes was the first to hold that, when we perceive, the representation need not resemble what it represents but should correspond to it. Descartes developed this ground-breaking, influential conception in his work on analytic geometry and then transferred it to his theory of perception. I trace the development of the idea in Descartes' early mathematical works; his articulation of it in *Rules for the Direction of the Mind*; his first suggestions there to apply this kind of representation-by-correspondence in the scientific inquiry of colours; and, finally, the transfer of the idea to the theory of perception in *The World*.

Keywords: René Descartes, representation, geometry, perception, colour.

1. Introduction

In my book, *Descartes' Philosophical Revolution: A Reassessment* (Ben-Yami 2015), I have shown in some detail that Descartes was the first thinker to hold a theory of representational perception with all the following characteristics:

- When we see colours, we are immediately aware of ideas of colour in our mind.
- The colour in the things we see causes our idea of colour.
- The idea of colour *represents* the colour in seen things.
- The colour in seen things does not *resemble* the idea of colour.
- The representation, when adequate, is so because it *corresponds* with what it represents.

(I focus here on vision, although the theory is supposed to apply to other sensory modalities as well). These characterisations of Descartes' view are all found in the scholarly literature and most are common in it, yet like so much else in this literature, some have been challenged. I provided in my book evidence for this interpretation of Descartes' theory and argued against some alternative ones (Ben-Yami 2015, chapter 2), and I shall assume it in what follows.

Theories of representational perception were common from antiquity onwards (Ben-Yami 2015, section 2.3, 33–43), yet Descartes' theory is original in several respects. For instance, Descartes is the first to hold that the representation of which we are directly aware is in the mind and not in the sense organs.

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This aspect of his theory, however, is one on which I shall not dwell in this paper. The innovative claim I shall discuss below is that the representation is adequate not through *resembling* what it represents but through having some sort of *correspondence* with it. This representation through correspondence, without resemblance, is true not only for the representation of colours by the ideas of colour in the mind, but also for their representation in the nervous system by various patterns of flow of animal spirits. I have provided in my book a historical survey to support my claim that the correspondence-without-resemblance view of representation was an innovation of Descartes' (section 2.3).

Descartes was aware of this innovative aspect of his theory of representation and of the consequent need to explain and justify it, something he therefore does at a few places in his writings. One place in which we find such a detailed explanation is the fourth discourse of his *Optics*. Descartes first explains why representation by means of resemblance is impossible in the case of vision:

> We must take care not to assume—as our philosophers commonly do—that in order to perceive, the soul must contemplate certain images transmitted by objects to the brain; or at any rate we must conceive the nature of these images in an entirely different manner from that of the philosophers. For since their conception of the images is confined to the requirement that they should *resemble* the objects they represent [*avoir de la ressemblance avec les objets qu'elles représentent*], the philosophers cannot possibly show us how the images can be formed by the objects, or how they can be received by the external sense organs and transmitted by the nerves to the brain (*Optics*, Discourse IV, AT 6, 112; CSM 1, 165; emphasis added).[1]

Having noted this, he continues to show, with an example taken from perspectival engravings, how an adequate representation sometimes *should not resemble* what it represents:

> Moreover, in accordance with the rule of perspective, [engravings] often represent circles by ovals better than by other circles, squares by rhombuses better than by other squares, and similarly for other shapes. Thus it often happens that in order to be more perfect as an image and to represent an object better, an engraving ought not to resemble it (*Optics*, Discourse IV, AT 6, 113; CSM 1, 165–66).

He concludes that this is the case with vision, where what is crucial is *correspondence* between representation and what is represented, and not resemblance:

> Now we must think of the images formed in our brain in just the same way, and note that the problem is to know simply how they can enable the soul to perceive all the various qualities of the objects *to which they correspond* [*les diverses qualités des objets auxquels elles se rapportent*]—not to know how they can *resemble* these objects (*Optics*, Discourse IV, AT 6, 113; CSM 1, 166; emphasis added).

[1] I almost always use existing translations, occasionally with minor revisions which I don't note.

His theory of representation in perception indeed involves correspondence without resemblance. This was a breakthrough in the understanding of representation generally and in the implementation of the idea in theories of perception, in philosophy as well as in physiology. From Descartes on, physiologists have developed models that explain how the nervous system preserves the *information* about the perceived objects, and did not try to explain how the colours of the things we see are *reproduced* in the brain.

A question that arises at this place is, why was *Descartes* the first to think of this kind of representation? One might of course claim that Descartes was a genius of sorts, and that a genius was needed to come up with this idea. History, however, has not been short of geniuses, and yet it was Descartes who first understood this possibility, so this response is insufficient. We need to understand what was special in Descartes' *circumstances* that made the idea of representation by correspondence accessible to him.

The answer I suggested in my book (section 3.3) was that Descartes transferred the idea of such a representation from analytic geometry to the theory of perception. In analytic geometry, algebraic entities represent geometric ones, and vice versa. This representation is of course devoid of any resemblance, while the different domains have corresponding structures that enable the representation. Accordingly, the idea of representation by correspondence was available to Descartes from his work in analytic geometry. In mathematics, work done during the last decades of the sixteenth century prepared the ground for the development of analytic geometry, which was indeed developed independently by Descartes and Fermat in the sixteen-twenties (Ben-Yami 2015, 241, note 20).

However, the treatment of the subject in my book left much work to be done. I did not trace there the development of Descartes' mathematical thought in a way which shows that the idea was available to him by the time he developed his theory of perception, and neither did I show in detail how the transfer of the idea from one domain to the other was accomplished. This is what I intend to do in this paper.

Descartes' mathematical thought developed gradually. We find him working on mathematical problems and methods quite early, in November 1618, following his meeting with Beeckman, but this does not mean that the developed techniques of his 1637 *Geometry*, their articulation and their application to complex problems occurred immediately. For instance, Descartes tried to solve Pappus's problem, which plays a central role in his *Geometry* and in demonstrating the power of his method, only in late 1631, after the Dutch mathematician Jacobus Golius had urged him to do so (Shea 1991, 60; Sasaki 2003, 3 and 206–7). Moreover, the stages of the development of Descartes' mathematical thought are controversial (see e.g., Rabouin 2010). His mature theory of perception, on the other hand, is already present in *The World*, which he started writing in 1629.[2] To defend the thesis of this paper it needs to be shown that

[2] By *The World* I refer to both treatises, *Light* and *Man*.

his understanding of representation by correspondence had been developed before that time.

The use of a technique and its clear conceptualisation do not necessarily arise together. In fact, one often acquires the former, albeit possibly to a limited degree, before the latter, and can describe it only through reflection on its existing use, a description that can then contribute to the technique's improvement. We should therefore expect that these stages might be found in Descartes' writings as well. Yet, as we shall see, both the technique and its articulation had been fully developed before Descartes started to work on *The World*.

Recourse to analytic geometry in order to explain the origin of the idea of representation by correspondence without resemblance might seem to introduce redundant complexities: hasn't *language* been available to Descartes, demonstrating this sort of representation? Moreover, doesn't Descartes *use* language to demonstrate this very idea of representation, already on the first pages of *The World* (AT 11, 4)?—I think that Descartes *did not* think of language as a representational medium, and that in the mentioned passage in *The World* he is arguing for a different point, namely, the possible lack of resemblance between cause and effect, as is also clearly seen in its later reworking in the *Principles of Philosophy* IV:197. Since I argued for this in detail in (Ben-Yami 2021), I shall not discuss it again in this paper.[3]

2. Earliest Mathematical Writings

2.1 *Cogitationes privatae*

The *Cogitationes privatae* or *Private Thoughts* of Descartes', which is known to us mainly through a copy made by Leibniz in June 1676, dates from 1619–1620[4] and contains the earliest mathematical writings of Descartes' (a few earlier ideas are mentioned in Beeckman's diary). I shall discuss here one problem that Descartes tries to solve in this work (AT 10, 234–35), which contains the most elaborate applications there of his technique of representing one domain by another.

Descartes asserts that he has found the solution of the equation $x^3 = 7x + 14$ and similar ones. In this context, finding the solution means, for him, specifying a geometric-mechanical procedure that yields a line whose length is the solution of the equation. From Euclid's day to Descartes', solving a problem meant finding a *construction* with the required properties (Shea 1991, 45). Accordingly, Descartes does not look for a method of arithmetical calculation that would yield the solution, as one might do today. Solving the equation thus involves the use of an instrument, and in this case, one invented by Des-

3 My 2021 paper supersedes my earlier discussion of this question in Ben-Yami 2015, 72–4.

4 For the history of the manuscript and of Leibniz's copy, both now lost, see Sasaki 2003, 109.

cartes, which he describes in the *Private Thoughts* and calls *a mesolabe compass* (AT 10, 238–39).[5]

The drawing in the *Private Thoughts* as well as the explanation there are not too clear, but luckily Descartes provides a more detailed description in the *Geometry,* accompanied by a more informative drawing. I shall therefore explain the working of the instrument by reference to them.

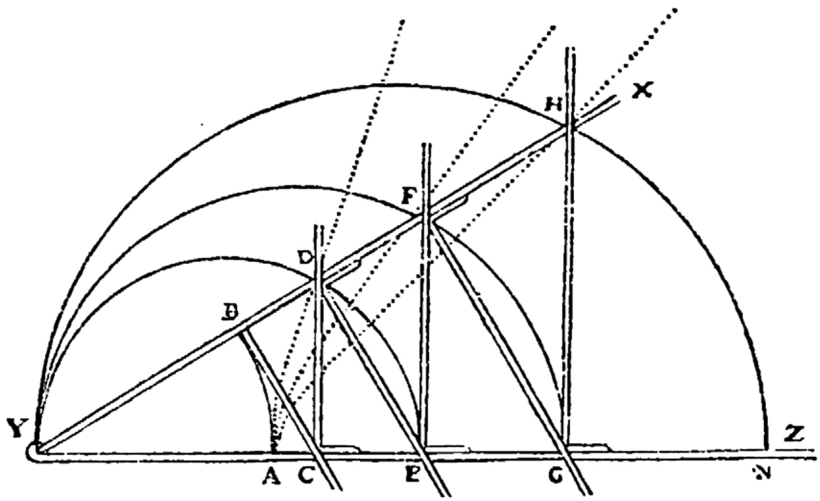

Figure 1 – Descartes' Mesolabe Compass (AT 6, 391; public domain).

The mesolabe is shown in Figure 1, taken from the *Geometry*. Its operation is as follows. While arm YZ remains stationary, arm YX can rotate around Y as axis. The ruler BC is fixed at a right angle relative to YX at point B. The rulers CD, EF and GH are at a right angle to YZ, and DE and FG at a right angle to YX, and they are all mobile. When we open arm YX,

> the ruler BC, which is joined at right angles to XY at point B, pushes the ruler CD toward Z; CD slides along YZ, always at right angles to it, and pushes DE, which slides along YZ, remaining parallel to BC. Then DE pushes EF, EF pushes FG, which pushes GH. And one can conceive of an infinity of other rulers, which are pushed consecutively in the same way, of which the ones always maintain the same angles with YX, the others with YZ (*Geometry*, Book II, AT 6, 391; translation, slightly altered, taken from Descartes 2001, 192).

The construction of the mesolabe makes the triangles YBC, YCD, YDE, and so on all similar.

[5] Descartes was led to the invention of his mesolabe through his work on music, contained in his *Compendium of Music*. When studying the work of Gioseffo Zarlino he came across Eratosthenes' mesolabe, which inspired his own. See Shea 1991, 38–40.

We can now turn back to the *Private Thoughts*. Descartes reduces there the equation, $x^3 = 7x + 14$ to the equation, $x^3/7 = x + 2$ and mistakenly claims that if he solves the equation, $x^3 = x + 2$ and then multiplies the solution by 7, he will find a solution to the former equation. He then introduces his mesolabe (Figure 2).

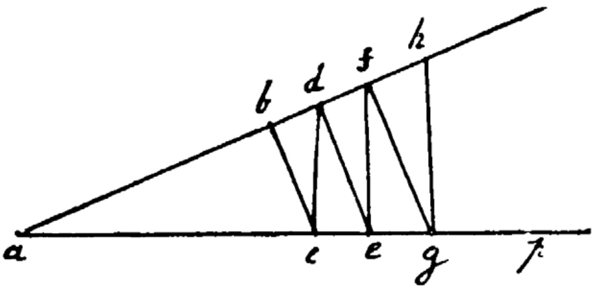

Figure 2 – The *Private Thoughts'* Mesolabe (AT 10, 234; public domain).

As we saw above, the triangles *abc*, *acd*, *ade*, etc. are similar. We therefore have:
$ab{:}ac = ac{:}ad = ad{:}ae = \ldots$
Setting $ab = 1$ and designating $ac = x$, we get:
$ab = 1$, $ac = x$, $ad = x^2$, $ae = x^3$
If we now open the mesolabe's arm *ah* until we get $ce = 2$, so that $ac + 2 = ae$, measuring the length *ac* will provide us with the solution of the equation, $x^3 = x + 2$.

What kind of representation do we witness in this case? First, numbers are represented by lines ($ab = 1$, $ac = x$ etc.). Moreover, addition of numbers is represented by addition of lines and ratios are represented through geometric relations, and in this way we obtain square numbers, cubes of numbers, etc., represented by lines (e.g., $ae = x^3$). Namely, already at this early stage of Descartes' mathematical thought, we find representation of items of the domain we investigate (numbers, algebra) by means of geometric entities through correspondence, without resemblance, and manipulation of the geometric entities leads to the solution of the algebraic problem.

2.2 *De solidorum elementis*

In 1676, Leibniz copied a manuscript which Clerselier held and that was later lost, which he titled *Progymnasmata de solidorum elementis excerpta ex manuscripto Cartesii* (*Preliminary Exercises on the Elements of Solids Extracted from a Manuscript of Descartes*). Leibniz's manuscript, which is dense and hard to read and comprehend, is still extant. By now there are two detailed and careful studies of it, which include a transcription and translations into English and French, by Pasquale Joseph Federico and Pierre Costabel (Federico 1982; Descartes 1987). The date of the manuscript has been debated, but it seems safe to date it to the years 1619–1623 (see Sasaki 2003, chapter 3, section 3D).

In the first part of this short work, Descartes tries to prove that there cannot be more than five regular polyhedrons. This has been proved already in antiquity, but by purely geometric considerations; the innovation in Descartes' approach is that he tries to do that by algebraic means. This aspect of his work makes it relevant to our interests here.

Descartes designates the number of solid angles by α, and the number of faces by a cossic symbol which I shall replace here with β. He then adduces various considerations and concludes that both $(2\alpha - 4)/\beta$ and $(2\beta - 4)/\alpha$ should be integers. A simple calculation then shows that there are exactly five solutions to the ordered pair (α, β): (4, 4), (6, 8), (8, 6), (12, 20) and (20, 12). These solutions yield the five regular polyhedrons.

The solution of this problem uses representation of geometric properties in an algebraic medium. The representational relations that are involved, as well as the manipulations needed to solve the problem, are more elementary than what we have seen in the *Private Thoughts* problem. However, the fact that now geometry is represented by algebra evinces a degree of abstraction in the approach to representation by correspondence: not only geometric entities do the representational work, but whichever medium that can serve to solve the problem addressed.

2.3 Descartes' "Old Algebra"

In a letter to Mersenne from early 1638, Descartes mentions a work to which he refers as his old *Algebra*, "ma vieille Algèbre" (AT 1, 501). The work is probably identical with a book Descartes showed Beeckman when they met in October 1628, the first meeting since they had parted in 1619. Later, in 1638, Descartes already thought that it was a work "not worth being seen" (AT 1, 501), having been superseded by his *Geometry*. However, it contained work from the mid-twenties, and as such represents an important stage in the development of his mathematical thought: later than the earliest works of 1619–1623 but still preceding the period of *The World* and *Geometry*. Moreover, Mersenne mentioned in his *Harmonicorum libri* (*Books of Harmony*: Mersenne 1636, 146–47) a proof that Descartes' work contained as one that Descartes had shown him in the summer of 1625. Accordingly, this proof, which I shall mention next, probably precedes also at least much of the work on *Rules for the Direction of the Mind*, which I consider in the next section. Other parts of the old *Algebra* may also be as early, but certainly precede the work on *The World*.

The old *Algebra* did not survive, but we learn about some of its contents from the reports of Mersenne and Beeckman. Two problems that Beeckman reports interest us here. Beeckman describes the first as, "It Is Demonstrated That One Can Find Two Mean Proportionals by Means of a Parabola" (AT 10, 342). Descartes finds the mean proportionals by intersecting a circle and a parabola. This is an advanced use of geometry to solve an arithmetical problem. I shall describe in more detail, however, the second problem, which demonstrates an even more advanced use of the representational technique.

Beeckman describes the second problem as follows:

With the help of a parabola to construct all solid problems by a general method. That M. Descartes in another place calls a universal secret to resolve all equations of third and fourth dimension by geometric lines (AT 10, 344; translation taken from Sasaki 2003, 172).

The equations Descartes discusses are of the form, $x^4 = \pm px^2 \pm qx \pm r$.[6] Descartes describes the construction given in Figure 3.

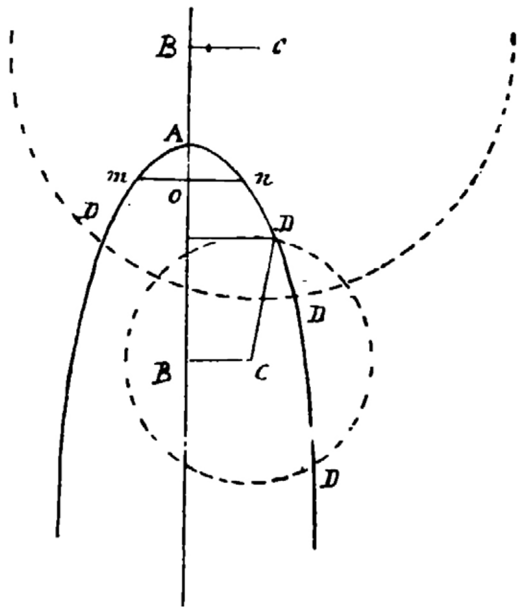

Figure 3 – Descartes' Construction for Fourth-degree Equations (AT 10, 345; public domain).

I shall consider only the case in which all signs are positive, namely, $x^4 = px^2 + qx + r$. The construction proceeds as follows. We draw a parabola with a vertical axis, vertex A as highest point and *latus rectum* 1. Take AB = (1 + p)/2 from A down along the axis. Next, take BC = q/2 perpendicular to the axis either to the right or to the left (to the right). Take a line segment of length = $\sqrt{}$ (CA^2 + r) and draw a circle with centre C and this segment as radius. The circle intersects the parabola at points D; draw perpendiculars from points D to the axis. These segments from the axis to D are the solutions; if D is at the same side of the axis as C, then the segment gives a positive root, while otherwise it gives a negative one.

6 I follow in my presentation Bos 2001, 256–57, where a modern proof of the correctness of the solution is also found. See also Shea 1991, 54–7.

This complex construction is doubtlessly a great achievement. Little wonder Descartes was highly pleased with his achievements, and with characteristic modesty told Beeckman

> that insofar as arithmetic and geometry were concerned, he had nothing more to discover; that is, in these branches during the past nine years he had made as much progress as was possible for the human mind (AT 10, 331, translation taken from Sasaki 2003, 159).

Irrespective of that, it is clear from the construction that by the mid-sixteen-twenties Descartes has made great progress in the technique that interests us: Complex entities of one domain are represented by those of another; in addition, the representation is through correspondence and without resemblance; the representing medium is again geometry; and complex manipulations in the representing medium track properties of the represented one, in this case algebraic equations, and in this way problems pertaining to the represented domain are solved.

3. *Rules for the Direction of the Mind*

Anything written on Descartes' *Rules for the Direction of the Mind* (*Regulae ad directionem ingenii*), and certainly any work that makes claims about the development of his thought, should be reconsidered now that the recently discovered purportedly early manuscript version of *Rules* has been published. However, this paper had been submitted and gone through revisions before the publication of that manuscript (April 2023), and the author could not therefore do that. In case that manuscript shows that important revisions or additions should be made to the analysis below, I hope to publish these later, at least as online material.[7]

3.1 The Method

Descartes worked on *Rules* from sometime in the mid-twenties until he moved to Holland in late 1628, leaving the work unfinished. Accordingly, the significant mathematical achievements discussed above antedate this work. The impression they left on Descartes is apparent in what *Rules* tries to develop: a scientific methodology based on the method that Descartes has been successfully applying in his mathematical work.

Rules 13 to 24 were supposed to discuss the method, but of these only rules 12 to 18 are developed, while rules 19 to 21 consist of titles alone, and the rest not written. However, the method of representing the object of research by means of geometric entities is clearly described.

Descartes' science is a mathematical science, dealing with quantities. All the examples he provides are from physics, which is also, apart from pure mathematics, the subject that he investigated in his earlier writings (e.g., *Private Thoughts*,

[7] The recent study of *Rules* by Tarek R. Dika (Dika 2023) was also published too late (March 2023) to be consulted for this work.

AT 10, 219 and following). The ideal of science to emerge from *Rules* is thus that of mathematical physics. This science deals with quantities, which according to Descartes should be represented in abstraction from their specific subject-matter:

> We can also see how, by following this Rule, we can abstract a problem, which is well understood, from every irrelevant conception and reduce it to such a form that we are no longer aware of dealing with this or that subject-matter but only with certain magnitudes in general and the comparison between them (*Rules*, Rule 13, AT 10, 431; CSM 1, 52).

Moreover, the magnitudes are to be represented by means of geometric entities. The title of Rule 14 is:

> The problem should be re-expressed in terms of the real extension of bodies and should be pictured in our imagination entirely by means of bare figures (AT 10, 438; CSM 1, 56).

These geometric entities are the preferred means of representation because

> it will be very useful if we transfer what we understand to hold for magnitudes in general to that species of magnitude which is most readily and distinctly depicted in our imagination. But [...] this species is the real extension of a body considered in abstraction from everything else about it save its having a shape. [...] Let us then take it as firmly settled that perfectly determinate problems present hardly any difficulty at all, save that of expressing proportions in the form of equalities, and also that everything in which we encounter just this difficulty can easily be, and ought to be, separated from every other subject and then expressed in terms of extension and figures (AT 10, 441; CSM 1, 58).

Descartes clearly transfers his mathematical technique to scientific enquiry generally. So much so that he next writes,

> At this point we should be delighted to come upon a reader favourably disposed towards arithmetic and geometry [...] For the Rules which I am about to expound are much more readily employed in the study of these sciences (where they are all that is needed) than in any other sort of problem (AT 10, 442; CSM 1, 58).

Still, while Descartes sees the method as clearly *exemplified* in mathematics, its use is far wider:

> These Rules are so useful in the pursuit of deeper wisdom that I have no hesitation in saying that this part of our method was designed not just for the sake of mathematical problems; our intention was, rather, that the mathematical problems should be studied almost exclusively for the sake of the excellent practice which they give us in the method (AT 10, 442, CSM 1, 59).

Accordingly, when writing *Rules* Descartes was not only in full mastery of his mathematical method but he also explains it clearly, and moreover generalises its applicability to all domains of scientific enquiry. It involves representation of *any* subject matter by means of geometric entities. Clearly, usually no resemblance exists in such representations, although the properties and relations of the represented correlate with the representing geometric properties.

Descartes' conceptualisation of his method in *Rules* agrees with that found about a decade later in his *Discourse on Method*:

> All the special sciences commonly called 'mathematics' [...] agree in considering nothing but the various relations or proportions that hold between their objects. And so I thought it best to examine only such proportions in general, supposing them to hold only between such items as would help me to know them more easily. At the same time, I would not restrict them to these items, so that I could apply them the better afterwards to whatever others they might fit. [...] I thought that in order the better to consider them separately I should suppose them to hold between lines [...] But in order to keep them in mind or understand several together, I thought it necessary to designate them by the briefest possible symbols. In this way I would take over all that is best in geometrical analysis and in algebra, using the one to correct all the defects of the other (*Discourse,* AT 6, 19–20, CSM 1, 120–21).

Unlike *Rules*, this later description of the method is supposed to apply only to the mathematical sciences, without claiming at this place that the method is applicable in all of science. And although the relations or proportions are here said to be represented only by lines, the practice of the *Geometry*, published together with the *Discourse*, shows that these lines are often used to construct more elaborate curves in order to achieve adequate representation of complex relations. We thus see that the methodology of *Rules* is that found in the mature description of the *Discourse*. Descartes of *Rules* is in full mastery of the representational technique of his later *Geometry*, as well as of its conceptualisation.

3.2 The Application in Perception

An important example in *Rules* of the application of the method is that to the study of perception, and more particularly of sight. I have argued in my book that while writing *Rules*, Descartes did not yet hold his later theory of the physical world as being pure extension but that he rather thought, following the Aristotelian tradition, that objective colour resembles our idea of colour (Ben-Yami 2015, 45; here and below I use "objective" in our contemporary sense, not in Descartes'). It follows that representation of colour and of the idea of colour by geometric figures is not through resemblance.

Descartes' discussion of the representation of colour occurs while discussing our cognitive powers (AT 10, 412–17). As with all other objects of scientific inquiry, it too, and all other qualitative sensory properties, should be represented by geometric figures. As Descartes writes later in the book,

> One thing can of course be said to be more or less white than another, one sound more or less sharp than another, and so on; but we cannot determine exactly whether the greater exceeds the lesser by a ratio of 2 to 1 or 3 to 1 unless we have recourse to a certain analogy with the extension of a body that has shape (AT 10, 441; CSM 1, 58).

Descartes should therefore provide a way of representing colours by shapes or figures. He thus asks us to "conceive of the difference between white, blue, red, etc. as being like the difference between the following figures or similar ones," as in Figure 4 (AT 10, 413; CSM 1, 41):

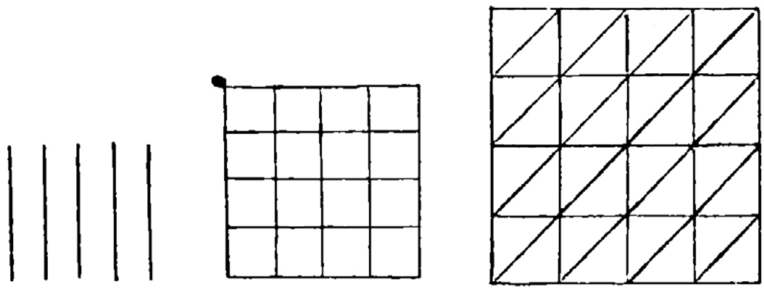

Figure 4 – Descartes' Representation of Colours in *Rules* (AT 10, 413; public domain).

Descartes does not explain at this place or anywhere else why he suggests *these* figures. Probably, the five vertical lines represent white, conceived of as the simplest, purest colour. But why should then qualitative blue be represented by a pattern of squares and qualitative red by the same pattern with diagonal lines added, and whether the increasing proportions of the drawings play any role in the representation, is hard to figure out. I am not familiar with any theory of colour in Descartes' writings or of his time that sheds any light on these representations. His later theory of objective colour in the purely extensional physical world is unrelated to these representations: colour is there the ratio between the pressure in the direction of propagation of light and the rotational pressure of the globules whose pressure is light (*Meteorology*, Discourse VIII, AT 6, 333–35; *Description of the Human Body*, AT 11, 255–56; letter to Mersenne, December 1638, AT 2, 468). This later theory allows each colour to be represented by two lines, one that represents the translational pressure and one that represents the rotational pressure relative to the translational one. It therefore makes the representation of red in *Rules* by squares with diagonals unnecessary and even meaningless. (This also shows that at the *Rules* stage, Descartes did not hold his later "geometric" theory of colour). Accordingly, Descartes' later theory does not help us understand his suggestion for the representation of colours in *Rules*.

Whatever the reasons for *Rules*' suggested scheme of representation of colours are, we have here a representation of qualitative, sensory qualities by geometric figures. This representation is supposed to be by means of some correspondence, obviously without resemblance, between the properties of the representing medium and what is represented. Accordingly, while writing *Rules*, motivated by his ideal of mathematical physics and consequent representational methodology, Descartes conceived of a systematic correspondence between colours and geometric figures and properties, which enables the one to represent the other.

4. From *Rules* to *The World*

When Descartes wrote the first few pages of *The World*, he already held the view that the ideas of light, colour and other sensory qualities do not resemble the things they are ideas of—the objective light, colour, and so on—a view he mentions there. Moreover, as is clear from later in that work, he also already held the view of the physical world found in Galileo's *The Assayer* (*Il saggiatore*, 1623), as being pure extension (I do not consider in this paper Descartes' reasons for adopting this view). This view enabled him to describe his physics as nothing but geometry (letter to Mersenne, 17 July 1638, AT 2, 268). In this geometric world, there is no place for the sensory qualities of which we are directly aware. Descartes had therefore to relocate them to something which is not material, or not purely material, namely to the immaterial mind, which is united in the living human being with a part of the brain (the pineal gland, called "gland H" in *Man*).

The idea of colour of which we are directly aware cannot therefore *resemble* its cause in the physical world. However, Descartes already had the conceptual resources to make the idea represent its physical cause adequately. By then, he had developed the concept of representation by correspondence and put it to much use, as we have seen in the previous sections when examining his earlier mathematical works. Accordingly, the lack of resemblance between the idea of colour and its purely "geometrical" cause is not in itself a reason to hold that an adequate representation of the cause—objective colour—is impossible. *Correspondence* between idea and *ideatum* is still possible.

In addition, although Descartes' favoured medium of representation has been geometric figures, he occasionally used algebra to represent geometric figures and properties, and by manipulating the algebraic representations solved geometrical problems: we saw this at work when we examined his *Elements of Solids*. Representing geometric entities is therefore something he had already done before he started working on *The World*.

Lastly, we saw that while writing *Rules*, Descartes suggested, for methodological reasons, representing colour by means of geometric figures. This kind of representation is achieved through a correspondence between the properties of the representing elements—geometric figures—and what they represent—colour. Namely, already at this stage Descartes conceived of a correspondence between colour and geometric entities.

Accordingly, Descartes had in his conceptual toolbox all the means he needed to develop a theory of representation through correspondence in perception. To achieve adequate representation within the framework of his new theory of perception, he just needed to reverse the *Rules*' relation between representation and represented. First, geometric properties are now turned into the thing being represented. Secondly, the ideas of colour, which are supposed to represent objective colour, can exhibit correspondence with geometric properties, as they did in *Rules*. Uniting these elements, we get Descartes' theory of representation in perception: the ideas of colour, these subjective sensory qualities, represent through correspondence objective colour, a property of Descartes' geometric physical world. The road to modern theories of perception has been opened.

That Descartes, while writing *The World*, thinks along the same lines (1) on the scientific representation of qualities by means of geometric entities, a representation of the kind we met with in *Rules*, and (2) on the representation of perceived reality by the nervous system and the soul or mind, is shown, among other things, by his terminology. I consider two kinds of representation he discusses.

Hearing, according to Descartes, is caused by "little blows with which the external air pushes against a certain very fine membrane stretched at the entrance to [cavities in the back of the ear]." The air behind the membrane is moved by these little blows and transmits its movement to fibres at the back of the ear. These connect to the brain and "will cause the soul [*donneront occasion à l'Ame*] to conceive the idea of sound." While a single blow produces only a dull noise, a sequence of such blows produces a sound, which the soul "will judge to be higher or lower depending on whether they follow one another slowly or quickly" (AT 11, 149–50; Descartes 1998, 122).

When several sounds are heard together, Descartes holds, they "will be harmonious or dissonant depending on the extent to which their relations are orderly, and on the extent to which the intervals between the blows making them up are equal" (AT 11, 150; Descartes 1998, 123). To explain that, Descartes uses the diagram given in Figure 5.[8]

In this diagram, lines A to H represent different sounds: a line represents a series of "blows," each represented by a notch, and the time between the blows is represented by the distance between notches: "the divisions of the lines A, B, C, D, E, F, G, H represent [*représentent*] the little blows that make up that number of different sounds." Since the distances between the blows on G and H are irregular, "[the sounds] represented by the lines G and H cannot be as smooth to the ear as the others." Moreover, given the ratio of the distances between the notches on lines A to F, "B must be considered to represent a sound an octave higher than A, C a fifth higher, D a fourth, E a major third, and F a full major tone" (AT 11, 150; Descartes 1998, 123). Descartes then continues to discuss relations of consonance and dissonance between the different sounds. The representation of percussions of air on the auditory nerves and their temporal relations by means of geometric figures is here used for the analysis of the character of sounds and the relations between them, in accordance with the methodology of mathematical physics we saw in *Rules*.

[8] I am using at this place the illustration from the edition of *Man* in Latin, *De homine*, published in 1662, two years before the publication of the original French version. Annie Bitbol-Hespériès has remarked, following Erik-Jan Bos, that the illustration in *De homine* is probably closer to the original one by Descartes and, following Rudolf Rasch in the Dutch translation of the book (Descartes 2011; reference from Bitbol-Hespériès), that it is more faithful to the text (Bitbol-Hespériès 2021, 157–58). My points, however, apply to the later illustration in the French edition of 1664 as well (Descartes 1664, 36), an illustration also used in AT 11, 150.

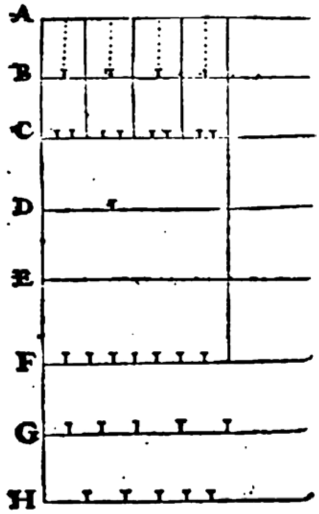

Figure 5 – Descartes' Representation of Sounds in *De homine* (Descartes 1662, 43; public domain)

Another use of "representation" occurs when Descartes discusses the representation on the retina of points at different distances, a representation rendered distinct by changing the shape of the lens, making it either flatter or more arched (AT 11, 156). In this case, some resemblance between the thing represented and its representation or image is still possible, yet this is not so in the following case. When discussing the formation of the ideas of objects that strike our sense (AT 11, 174–76), Descartes describes how light rays coming from an object press on optic nerves ending at the back of the eye while tracing there a figure of the object. The valves of these optic nerves open at their other ends, in front of the pineal gland, and consequently animal spirits from corresponding specific points on the pineal gland flow into these nerves. In this way, "that figure is traced on the surface of the gland depending on the ways in which the spirits issue from [these] points." The figures traced by the spirits on the surface of the pineal gland are the ideas, namely, "the forms or images which [...] the rational soul will consider directly when it imagines some object or senses it" (AT 11, 176–86; Descartes 1998, 149). And this pattern of spirit flow from the surface of the pineal gland represents all that we perceive:

> And note that by figure I mean not only things that somehow represent [*représentent*] the position of the edges and surfaces of objects, but also anything which, as I said above, can give the soul occasion to sense movement, size, distance, colours, sounds, smells, and other such qualities (AT 11, 176; Descartes 1998, 149).

Descartes emphasises that this figure, determined by spirits' pattern of flow, represents not only the *figures* of objects ("the position of the edges and surfaces"), but other diverse characteristics of the material world as well, such as movement, distance, smells, and more: it is important to him that his reader realise that representation can be of things it does not resemble at all. It can be achieved both by geometric figures representing sound, for scientific purposes, as we saw above, and by patterns of spirit flow, representing diverse properties of perceived objects.

The parallel conceptualisation of representation in mathematics and in perception is shown also by the talk on how the representation *corresponds* or *relates*—*se rapporter*—to what it represents. The first marginal heading in the *Geometry* reads, "How the calculations of arithmetic correspond to the operations of geometry" (AT 6, 369). And later, Descartes notes:

> The scruples that the ancients had about using the terms of arithmetic in geometry, which could only proceed from the fact that they did not see sufficiently clearly their correspondence, caused much obscurity and awkwardness in the way they explained themselves (AT 6, 378).

And similar formulations occur when discussing perception. When we look at an object directed a certain way, the soul will be able to tell how it is positioned because the nerves affected by the light coming from it will trace at the place in the brain from which they originate a figure which will correspond exactly ("se rapportera exactement") to it, and consequently a corresponding figure will be traced on the pineal gland (AT 11, 159 and 175–76). Correspondence with the object remembered is also used to explain memory (AT 11, 178). And generally, we should assume

> that each tiny tube on the inside surface of the brain corresponds to a bodily part, and that each point on the surface of gland H corresponds to a direction in which these parts can be turned: in this way, the movements of these parts and the ideas of them can cause one another in a reciprocal fashion (AT 11, 182; Descartes 1998, 154–55; cf. AT 11, 183).

Descartes gives additional detail on these pages of *Man* on how patterns of flow of animal spirits represent by correspondence the images we perceive and remember.

To recapitulate: we have seen that Descartes developed and employed the idea of representation by correspondence without resemblance in his mathematical work; that a little later (*Rules*) he thought of applying it to the study of perception for the purpose of mathematical physics; and that he then transferred it to perception itself, once his view of material reality as pure extension had been developed (*The World*). The terminology he uses also shows the related conceptualisations of the two domains. Accordingly, the idea was most likely transferred by Descartes from his mathematical thought to his thought on perception.

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