5. Conclusion and future development

In this Thesis, we presented novel (adaptive) fitting methods with THB-splines for the (re-)construction of highly accurate CAD models from input point clouds. We combined CAGD approaches with DL technology to produce robust, automatic, and efficient fitting schemes.

In Chapter 1, we collected a selection of preliminaries notions on B-splines, THB-splines and NNs, with special focus on CNNs and GCNs.

In Chapter 2, we reviewed interpolation and least squares approximation schemes and provided a new general formulation for reweighted least squares as a convex combination of certain interpolants. Furthermore, we exploited the weights for spline fitting problems, also in the case of adaptive THB-spline constructions. We proposed a strategy to automatically update the weights within the fitting scheme, either to emphasize data marked as sharp features or to smoothen data marked as corrupted [57]. The two-stage hierarchical QI scheme for the approximation of scattered dataset using THB-splines [14, 15] was then revisited. We modified the first stage of the scheme by introducing local B-spline approximations to handle distribution with varying density of points. This choice improved the performance of the existing scheme by increasing the accuracy of the model and simultaneously reducing the computational costs. The advantages of this choice was proven in the numerical examples, where the reconstruction of industrial geometries was addressed [17].

In Chapter 3, we proposed novel data driven models both for gridded and scattered point cloud parameterization. These models were characterized by suitable NNs architectures based on convolutional operators, defined on the considered domain. The PARCNN model [32] was based on a *pure* CNN architecture, i. e. consisting only of convolutional blocks. This choice was made to take advantage of the locality of convolutional operators in order to be able to support variable input sizes without any additional effort and/or pre- or post-processing of the data. To overcome the limitation of CNNs to process only data with a grid-like topology, we then considered GCNs to address the parameterization learning problem of *scattered* data. PARGCN [60] processed data with a *graph* structure, corresponding to the radius neighbours graph of the input scattered point cloud. Subsequently, we devised BIDGCN [58], a new GCN architecture for the parameterization

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of scattered data that takes into account boundary conditions in addition to the standard vertex features of the discrete surface.

All the proposed methods were agnostic to the size of the input point cloud, were robust to noise, and generalized to point clouds different from the ones used during the training phase. They outperformed both closed form, heuristic and data-driven parameterization choices and produced high-quality parameterizations for (TH)B-spline reconstruction schemes. In addition, BIDGCN, once trained, was computationally more efficient than the classical meshless parameterization methods.

In Chapter 4, we introduced novel adaptive fitting schemes with moving parameterization and THB-splines, based on the optimization of different error metrics [61, 59]. The first strategy to move the parameters, consisted of enriching the adaptive approximating loop with the PC routine [75]. As concerns the control points, we proposed different diverse update rules, which bring to the development of adaptive A-PDM, A-TDM, and A-HDM. In addition, we exploited the introduction of PC within the adaptive hierarchical QI scheme presented in Chapter 2. The second strategy to move the parameters consisted of addressing the parameterization problem together with the computation of the control points in the first step of the adaptive loop. This could be achieved by solving a non-linear joint optimization problem, J-PDM, which simultaneously computed the optimal parameter sites for the input point cloud and the optimal control points for the approximating surface. With this method, we avoided the need of solving a linear system of equations and performing PC at every adaptive iteration.

Our study revealed that, independently from the chosen strategy, addressing moving the parameterization within the adaptive loop could improve the fitting results while also reducing the number of degrees of freedom required to achieve a certain accuracy. This technique could lead to earlier termination of the adaptive process, thus providing more compact models with less refinement levels, being at the same time more accurate.

As concerns the parameterization of scattered point clouds, the methods in this Thesis assume the data to be already partitioned between interior and boundary points. In order to fully automate the point cloud reconstruction, a future research direction consists in developing new data-driven techniques to address the point cloud boundary detection problem, see e. g., [109, 72]. It is also of interest to develop new data-driven parameterization methods that learn quasi-conformal surface parameterization and reduce possible geometric distortion [25]. Finally, also domain parameterization could be enhanced by the development of suitable learning techniques, see e. g., [169].

As concerns fitting schemes, it is of interest to extend the methods presented in this Thesis to different adaptive spline functions e. g., [103, 81, 16]. Moreover, to design a full CAD model, multi-patch constructions need to be addressed. Consequently, it is of particular interest to develop multi-patch fitting schemes, see e. g., [115].