Chapter 2

Theory of Games and Formal Political Theory: Before Riker

This chapter will provide a general discussion of the development of the Theory of Games and of some formal theories of politics in the Fifties.

The broader climate of transformation in American social science framed the development of these theories. Whereas the first chapter discussed the so-called "Behavioral Revolution," which radically reshaped American political science from the Fifties onward, the following pages will also show how economic analysis addressed political issues, such as voting behavior, through formal modeling. Previously it has been highlighted the pivotal role of *Theory of Games and Economic Behavior* in shaping the mathematical outlook of postwar economics. Now it will be discussed how the theory of games moved across domains other than economics.

Accordingly, this story is about political science and the history of Game Theory. For Game Theory, its cross-disciplinary use is far from unproblematic and raises broader questions about its own evolution. Indeed, the delayed acceptance of Game Theory into mainstream economic theory dates only to the early Eighties. Moreover, what ultimately entered economists' toolbox was John Nash's approach (together with major refinements by John Harsanyi, Reinhardt Selten, Robert Aumann, Robert Wilson, David Kreps, among others), rather than the theory von Neumann and Morgenstern presented in the Forties. Yet the first attempts to use game theory to address social and political issues in the Fifties drew on both approaches.

American political scientist William Harrison Riker was a crucial figure in the history of how game theory entered political science because he did not merely adopt game-theoretic notions. Rather, the theory of games also served his methodological concerns about political science. On this point, Riker was adamant. In the first chapter of his 1962 work, *The Theory of Political Coalitions*, he wrote: "the main hope for a genuine science of politics lies in the discovery and use of an adequate model of political behavior." (Riker 1962b, p. 9). Such a model was explicitly game-theoretic. Thus, rational choice and mathematical modeling matched the expectations of a scholar like Riker, who was searching for new methods of political analysis.

Riker's efforts also paralleled related developments in economics. In the Fifties, scholars such as Kenneth Arrow, Duncan Black, Anthony Downs ,

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and Martin Shubik (with Lloyd Shapley), as well as James M. Buchanan and Gordon Tullock, produced formal analyses of collective choice, power, and voting, and new research fields such as "Social Choice" and "Public Choice" took shape. Although many of these works were not, strictly speaking, game-theoretic, they relied on the same behavioral model—often grouped under the label "Rational Choice Theory", that flourished in the wake of von Neumann and Morgenstern's 1944 work.

From this perspective, Game Theory plays a pivotal role. It influenced economics and the social sciences in two related ways. On the one hand, it provided the first fully consistent mathematical model of individual behavior (through von Neumann's work on utility under uncertainty), shaping subsequent models of rational choice. On the other hand, it formalized strategic behavior and offered early models of interaction. More generally, both developments affected the discourse on rationality and rational decision-making across the social sciences.

The first section of this chapter provides a somewhat detailed review of analytical aspects of the theory of games, focusing mainly on von Neumann and Morgenstern's framework. The second section discusses results in social choice (Arrow), analyses of voting and elections (Black and Downs), and their relationship with game-theoretic ideas.

2.1 The "Theory of Games and Economic Behavior"

Many works (starting from Weintraub 1992) have sought to reconstruct and discuss the indisputable impact of Game Theory on contemporary (post-Eighties) economics. Robert Leonard (Leonard 2010) showed how the concept emerged in the first half of the twentieth century and how it was embedded in the intellectual climate surrounding formalism and axiomatization in mathematics. In particular, the mathematician Ernst Zermelo's work on applying set theory to chess (1912) influenced the Hungary-born polymath and mathematical genius John von Neumann.

A second stream of influence, also discussed in Leonard's work, came from debates on the nature of the social sciences, and, more in general, on scientific knowledge, which animated the Viennese intellectual milieu until the Anschluss (1938). In particular, the *Theory of Games* reflected the concerns and aspirations of its creators, the Austrian economist Oskar Morgenstern and von Neumann. Before leaving Austria in 1938, Morgenstern was among the most active scholars engaged in such discussions. In 1928, von Neumann published a mathematical result defining rational behavior in a “game” with two players and opposed interests (Neumann 1928). The two met at Princeton in early 1939. Morgenstern arrived there in 1938. Von Neumann had joined the Institute for Advanced Study shortly after its establishment (1932), although he continued to spend research time in Europe at least until political circumstances made this impossible.[1] Jointly,

[1] Von Neumann was Jewish by origin. He was able to bring his and his wife's families into the U.S.A. in 1939. Morgenstern was not Jewish, and although his political attitudes were less libertarian than those of other members of the circle of Ludwig

they published in 1944 *Theory of Games and Economic Behavior* (Neumann and Morgenstern 1944).

Leonard's historical work explores the invention of Game Theory. Other scholars have focused on different issues, most notably the place that the original conception of Game Theory, as articulated in the 1944 founding text, occupies in the broader history of twentieth-century neoclassical economics. (Giocoli 2003b)

One striking feature of Game Theorys's history is that, although its importance for postwar economics cannot be denied, its influence on the discipline differed sharply from von Neumann's and especially Morgenstern's expectations, and its adoption was slower than they had hoped. What the postwar generation of young economists found in the 1944 text was, on the one hand, a concise yet detailed mathematical toolbox relevant to optimization theory and, on the other, a rigorous characterization of rational utility under uncertainty. These, especially the latter, filled a major gap and generated a large literature from the Fifties onward. Yet even a quick look at *Theory of Games*'s table of contents suggests that these topics do not occupy the central place in the book.[2] Much of the work is devoted to strategic situations involving many players (more than two) and allowing for agreement and coalition formation. This analysis builds on von Neumann's pivotal 1928 demonstration of an equilibrium solution for a two-person zero-sum game, the Minimax Theorem (for which *Theory of Games* also provides a refined proof). Yet the most distinctive contribution of the book lies in its analysis of coalition formation. From this standpoint, much of *Theory of Games* differs substantially from what later became standard in game theory textbooks.

The reasons for the delayed rise of Game Theory in economics are multiple. They largely concern features of von Neumann and Morgenstern's framework itself. For decades, it was unclear what Game Theory was meant to accomplish for economic theory and how it could do so. It addressed strategic behavior, but the behavioral foundations of coalitional games (the kind emphasized in *Theory of Games*) were often regarded as unclear. Only the theory of two-person games of pure opposition (two-person zero-sum games) offered a comparatively well-defined framework.[3] This raised a basic question: what is the purpose of a theory of coalitions? It is difficult to treat it as prescriptive, since complex real-world situations rarely fit a well-specified game structure. For similar reasons, it also seemed hard to treat it as predictive.[4]

von Mises (to which Morgenstern belonged in the Twenties), he still opposed fascist policies. Therefore, in 1938, when spending a research period in the U.S.A., he decided not to return to Austria. Subsequently, he obtained a full position at Princeton. Leonard 2010 contains these and other biographical details. For von Neumann's intellectual and scientific biography: Israel and Gasca 2009

2 As a matter of example, the two authors added the detailed axiomatic proof of their utility function only in the second edition of *Theory of Games*, published in 1947

3 Henceforth 2ZPSG

4 One interesting page of the development of Game Theory in economics concerns the employment of a cooperative solution, the "Core," to prove the existence of the General Economic Equilibrium. The Core is the set of all undominated imputations

From the standpoint of technical economics, Game Theory, both von Neumann and Morgenstern's original work and the refinements developed by Princeton scholars such as Nash, Tucker, Kuhn, and Shapley contributed substantially to economic theory in terms of tools, solution techniques, and theorems. For example, most textbooks of mathematical economics through the Seventies included at least a brief discussion of the Minimax theorem, particularly after it was shown in the late Forties that it corresponds to a linear optimization problem. Another important example concerns fixed-point theorems, a topological tool first employed by von Neumann in the only work he devoted explicitly to economic theoy (Neumann 1945).[5], then by Nash in his dissertation, and later by Arrow and Debreu and by McKenzie in their proofs of the existence of general equilibrium.[6]

Nonetheless, Game Theory remained peripheral in economics until the Eighties. Economists widely employed Rational Choice under certainty and uncertainty, and most theoretical problems were framed in terms of optimization. Moreover, the discipline increasingly adopted mathematical modeling. Yet only a small group of scholars took Game Theory seriously and remained committed to its core conceptual and theoretical problems.[7] This situation persisted until the late Seventies and early Eighties. Only then, partly due to diminishing enthusiasm for general equilibrium theory and the emergence of powerful ways to model richer economic environments as games (e.g., alternative models of competition), did Game Theory become central within economics.[8]

This simplified narrative broadly captures the case of economics. How-

(see below). Martin Shubik showed that it is part of the "contract curve" in the Edgeworth box, familiar from any microeconomics textbook. In a nutshell, given a group of households, some allocations of goods are proposed. The coalitions of households form to support or block the proposed allocations. A coalition blocks an allocation if another assignment is Pareto superior for its members. Therefore, the Core of the economy is the set of feasible allocations that are not blocked by any coalition. Many "existence papers" proving that the competitive equilibrium allocation is in the Core appeared in the Sixties by scholars like Debreu, Shapley, Shubik, and Herbert Scarf. (See Cogliano 2019). However, since these results did not seem ostensibly better than the existence theorems proved by more traditional methods (like the Arrow-Debreu model), they did not enhance game theory's role in economic theory. On the contrary, to many scholars, it definitely showed that Game Theory was only a particular case of the general approach.

5 A history of this contribution is contained in Weintraub 1983

6 For the history and comment on these fixed-point techniques see Giocoli 2003a

7 Many personal reminiscences and anecdotes of economists show how limited the study of Game Theory was even in those graduate programs extremely committed to mathematical mode, like MIT. See Fudenberg and Levine 2016; Roth and Wilson 2019

8 For a historical-review paper quite challenging of the adequacy of Game Theory in providing an effective solution to Industrial Organization problems, see: Fisher 1989. Similarly, Abu Turab Rizvi contended that Game Theory was able to "rescue" economic theory from the impasse of the General Equilibrium Theory (due to the impossibility of proving that competitive equilibria display "uniqueness" and "stability," other than existence. The main reason lies in the arbitrariness of the models' assumptions, with special regard to the role of information (Ingrao and Israel 1987; Rizvi 1994).

ever, Game Theory was never confined to that discipline.[9]

Although Game Theory was deeply embedded in neoclassical economics' theoretical problems, like equilibrium, rationality or prediction, it was also, in important respects, more "social" and less narrowly "economic" than is often presumed. This connects to another well-known aspect of Game Theory's history: military strategists, international relations specialists, government officials, and scientists used game-theoretic ideas to address urgent Cold War problems.

This is, in some respects, a curious development. The initial limited uptake of Game Theory in economics was tied to a persistent question: what could modeling a game explain about the real world that existing models could not? The difficulty of answering this question pushed many economists away from the theory. Yet researchers concerned with more concrete and high-stakes problems than, for example, modeling market-clearing manifolds, often found the theory useful.

One plausible answer is the following: Von Neumann and Morgenstern's theory of n-person games did not capture the full range of issues involved in general equilibrium or individual rational action. Non-cooperative Game Theory proved more serviceable to many researchers, but it was not easy to integrate with existing rational choice approaches.[10] By contrast, when the number of players is small (for example, two), simpler uses of Game Theory, such as representing strategies and outcomes in matrix form, can be advantageous. Since much of two-person game theory was already developed in the Fifties, it is reasonable to infer that Game Theory could be functional for certain political phenomena even when economic problems were, at the time, more effectively treated using the dominant tools of economics in the Fifties and Sixties.

A different, more sociological answer would emphasize the distinctive intellectual commitments of the institutions that shaped the so-called "Cold War rationality" (see Erickson et al. 2015; Erickson 2015).

When one thinks about Game Theory in political science, the immediate association is often a strangelovesque connection to Cold War strategy, nuclear deterrence, and related topics. Less well known is that Game Theory entered political science through another route, namely through Riker's scholarly activities. Unlike many international relations theorists or "Cold War warriors," Riker sought to establish an entire subfield of political research in which the Theory of Games was not merely an instrument but the core of theoretical development. The historian Sonja Amadae and the political scientist Bruce Bueno de Mesquita wrote that: "it was Riker that bestowed on the game theory the promise of new life after RAND defense strategists concluded it had little merit for studying warfare and before

9 This point is familiar to historians of ideas and sociologists of science who emphasize how economic techniques crossed disciplinary boundaries. It is not the purpose here to engage fully with that literature. Something will be said in the conclusions and the text when dealing with specific issues but only to note that, in the case of Game Theory, the story is less linear than is often assumed.

10 Especially because Game Theory was by far less intuitive in its theoretical premises. See the following paragraph

economists grasped its promise for grounding a new mathematics of the market." (Amadae and Mesquita 1999, p. 278). This statement is perhaps somewhat too generous. Game Theory never ceased to be studied at places such as RAND, even during the height of general equilibrium theory in the Sixties and Seventies. Moreover, among economists, the reception of Riker's game-theoretic work was close to zero, at least in the Sixties.

Nevertheless, Riker's role in the cross-fertilization between game theory and other social sciences cannot be ignored. He drew on elements of von Neumann and Morgenstern's original framework—the very approach that many economists had set aside and that was not central in international relations scholarship. He did so with unusual commitment and with strong confidence in Game Theory's capacity not only to model but also to generate predictions about political outcomes. The institutional and intellectual consequences of Riker's advocacy became especially visible after the Sixties, when, scholars like Richard McKelvey and Peter Ordeshook (among others) developed a type of Political Science that followed Economics at the heights of mathematical modeling. Both possessed a Ph.D. in the graduate Political Science program Riker established at the University of Rochester, starting in the mid-Sixties.[11]

2.1.1 "The best way of playing a game": rationality and taxonomies of games

At the core of Game Theory lie the notions of "game" and "rationality." The idea of developing an algorithmic procedure to determine the best way of behaving in situations such as parlor games attracted some attention among mathematicians in the first half of the XXth century. Chess offers the best-known example of a pure-opposition game.[12] At the same time, economists were struggling to provide a theory of human behavior that could clarify the rationale behind economic choices. In the Thirties, as seen, debates over equilibrium, perfect foresight, and how to model economic phenomena were particularly intense.

In 1928, von Neumann wrote the first mathematical paper on Game Theory (which he labeled *Gesellschaftsspiele*, i.e., games of society). In that work, he studied two-player games of pure opposition, showing how rational players should behave in such settings: the "Minimax Theorem." (Neumann 1928) In his joint work with Morgenstern, he extended this approach to n-person games, where a natural opposition of interests arises not between individual players but between groups of them, or "coalitions." In the 1944 work, the problem of rational behavior is the starting point from which the authors develop their analysis. Accordingly, beyond the discussion in

11 These topics will be part of this dissertation's final part.

12 In the game-theoretic jargon, chess is 2PZSG, finite, and with perfect information. "Finite" means that there is a finite series of moves. "Perfect information" means that each player knows the opponent's moves. Before Game Theory was invented, German mathematician Zermelo provided the valuable insight that such a game always had a solution. Princeton mathematician Harold W. Kuhn proved Zermelo's intuition, showing that a 2PZSG, finite and perfect information, has a Nash Equilibrium for pure strategies (see Kuhn and Tucker 1953).

the first (verbal) chapter, they offered a mathematical characterization of individual rational action in two ways: first, by providing an axiomatic treatment of individual utility—one of the long-standing issues in the economic theory of their time (Moscati 2018)[13]; and second, by reformulating the minimax more accessibly (while also introducing the mathematical notions needed to understand the general result).

Starting with the former, they axiomatically derived an "Expected Utility Function," namely a real-valued function of the form:

$$\sum_{i=1}^{n} p_i u_j(x) \quad \text{for} \quad i, j = 1 \ldots n,$$

where p denotes the probabilities associated with each possible outcome $u(x)$. That is, for each individual, utility is computed as the sum of each possible disjoint event multiplied by the probability of its occurrence.[14] After their work, it became customary for economists to define rationality as consistency with axioms concerning properties such as orderings and preorderings between alternatives (or preferences), and to use these properties to construct continuous and differentiable utility functions.[15]

The reception of von Neumann and Morgenstern's utility function was immediate. It paved the way for a long sequence of debates on the empirical plausibility of utility theories (for instance, using psychology and laboratory experiments) as well as on the formal characterization of utility theory and rational choice. However, within von Neumann and Morgenstern's Game Theory, utility theory played a limited role. It mainly served to justify

13 The two authors made a decisive step forward in the debate between the "ordinalist view" of utility and the "cardinalist view." After the so-called marginalist revolution, this controversy started in the last decades of the XIXth century and lasted until the Thirties. In that decade, the idea that a consistent approach for representing a utility function required only that such a function was unique up to any monotonic increasing transformation revolutionized Consumer Theory (Hicks and Allen 1934). However, dealing with the issue of choice under uncertainty, which involves probability, von Neumann built a cardinal mathematical function, i.e., it is unique up only to a class of transformation, namely the positive linear ones. He showed precisely where the differences between choices under uncertainty and choice under certainty rested.

14 A verbal discussion, with axioms but without mathematical proof of this function, is contained in the first chapter of the work.(Neumann and Morgenstern 1944, 24 et ss.)

15 Arrow described Rational Choice as the choice that satisfies the properties of connectedness and transitivity (see Arrow 1951b, 17 et ss.). Debreu's classical proof of the existence of a utility function on a compact space if the preferences are continuous adopts a similar axiomatic approach (see Debreu 1959). The review of the enormous debate around the concept of expected utility and the meaning of "cardinality," which involved not only economists and decisions theorists but also psychologists, is beyond the scope of these pages. From a historical point of view, the most recent and most comprehensive review is by Moscati: Moscati 2018. Otherwise, the discussions around Expected Utility Theory, its properties, and developments (like Leonard Savage's theory, and Aumann-Anscombe's, can be found in any advanced microeconomics textbook

the notion of payoffs as quantities representable by a single number and, therefore, to ground strategic action. To identify the best course of action in a game, a stronger characterization of each player's rational behavior was needed—namely, that provided by the Minimax. This result, originally demonstrated by von Neumann in 1928 and reprised in the 1944 work, states that in two-player situations of pure opposition (2PZSG), each player should choose the strategy that minimizes the maximum payoff the opponent could obtain. (Neumann and Morgenstern 1944, 85 et ss. Neumann 1928)

The Minimax is a clear mathematical result that relies on a fundamental property of the theory of functions: the existence of "saddle points." Namely, $\max_x \min_y F(x,y) = \min_y \max_x F(x,y)$ holds when there exists a saddle point (x_0, y_0) satisfying $F(x, y_0) = \max_x F(x,y)$ and $F(x_0, y) = \min_y F(x,y)$. Von Neumann employs this result to solve a version of the two-person game in which players adopt only "pure" strategies (there is no chance) and interests are perfectly opposed (the game is "zero-sum"). The "zero-sum condition" is: $F_1(s_1, s_2) + F_2(s_1, s_2) = 0$, from which $F(s_1, s_2) = -F(s_1, s_2)$.[16] Player 1 wants to maximize $F(s_1, s_2)$, while player 2 wants to maximize $-F(s_1, s_2)$, which is equivalent to minimizing $F(s_1, s_2)$. Therefore, each player's behavior can be written as:

$$\max F(s_1, s_2) = \min F(s_1, s_2)$$

Conceptually, the difficulty is that each player controls only her own strategy, but the opponent's behavior is also payoff-determining. Von Neumann addresses this problem by dividing the game into two subgames: a "Minorant Game," where player 1 chooses his strategy before player 2, and a "Majorant Game," where player 2 chooses his strategy before player 1. Starting with the first, the "good way" to play for player 1 is to choose a strategy that maximizes the function $\min_{s_2} F(s_1, s_2)$. Indeed, player 2, acting after player 1, will choose an s_2 that minimizes $F(s_1, s_2)$, i.e., will deliver $\min_{s_2} F(s_1, s_2)$. Therefore, for player 1, the value of the game is: $\max_{s_1} \min_{s_2} F(s_1, s_2) = v_1$

A symmetric argument holds for player 2 (the "Majorant Game"), except that he acts after player 1. Knowing that player 1 will maximize his payoff, player 2 chooses a minimizing strategy. If v_1 and v_2 are equal, then a saddle point exists for $F(s_1, s_2)$ and the game has a solution. This result relies on the existence of a pair of strategies that are saddle points. Of course, their existence is not automatically guaranteed for pure strategies. Von Neumann showed that it is guaranteed when one allows mixed strategies (where players choose probabilities over strategies).[17]

This conclusion, which avoids reference to players' psychology, offers a clear prescriptive justification for action and an objective criterion for what counts as rational strategic choice. For this reason, many scholars

16 Note that x and y are now s_1 and s_2, that is, player 1's strategy and player 2's strategy. I am using a different, and simpler notation than that used by von Neumann.

17 Note that the pure strategy is a particular case of the mixed one, i.e., the case when $p = 1$

have linked the result, and the proof offered in the 1944 work, to von Neumann's broader aims in developing Game Theory. Giocoli, for instance, argues that von Neumann was especially interested in the prescriptive content of rationality, a point he connects to this author's preference for a "direct proof" in their work.[18] Leonard instead frames this interest in von Neumann's concerns about the disruption of European political and social order, threatened by Communism and Fascism (Giocoli 2003a; Leonard 2010).

The normative orientation of von Neumann's concept of rationality is apparent in the formula he and Morgenstern used in their work: the "best way of playing a game." The two authors devote substantial space in the first chapter to clarifying what rational behavior means; yet, ultimately, they rely on the mathematical force of the Minimax. They attribute the absence of a satisfactory analysis of rational behavior in the social sciences to "the failure to develop and apply suitable mathematical methods to the problem." (Neumann and Morgenstern 1944, p. 11)[19]

This point deserves further elaboration. A rational individual, in the simplest case (the classical "Robinson Crusoe"), seeks to obtain the maximum of something (e.g., utility or money). However, this "maximum problem" takes on new features in social settings, which involve a "disconcerting mixture of several conflicting maximum problems." In the authors' words: "He too tries to obtain an optimum result. But in order to achieve this, he must enter into relations of exchange with others. If two or more persons exchange goods with each other, then the result for each one will depend in general not merely upon his own actions but on those of the others as well. Thus each participant attempts to maximize a function of which he does not control all variables." (Neumann and Morgenstern 1944, p. 11) Characterizing rational behavior in a positive sense is therefore difficult because rational decision-making requires forming expectations about what others will do. Morgenstern had already argued in the Thirties that this can generate paradoxes that undermine equilibrium notions premised on perfect foresight (Morgenstern 1976a). By contrast, the minimax breaks this regress by specifying each player's rational strategy when the opponent is also rational.

As a behavioral criterion, however, the Minimax was subject to criticism, especially for being overly defensive and thus weakening von Neumann's aspiration to provide a substantive normative justification of rational choice

18 Giocoli summarises very clearly the difference between the two approaches: an "indirect proof" gives only necessary conditions, that is, if an equilibrium exists, then the theory is consistent. Instead, the direct method gives sufficient conditions, paving the way for a better defined prescriptive characterization of strategic rationality, and entails normative value. It could also explain von Neumann's well-known coolness toward Nash's solution, namely the "Nash Equilibrium." Indeed, the latter has far less substantive content than the "minimax": only the Nash Equilibrium is the game's solution, and it does not matter how it is reached or why people pursue an equilibrium strategy.

19 Morgenstern is referring to having rested mainly on optimization theory using calculus instead of axiomatic treatment. However, the second does not exclude the first

(how and why agents are rational). The Minimax secures for each player a payoff regardless of what the opponent does: assuming the opponent is rational, choosing the strategy that minimizes the maximum the opponent can obtain delivers a security payoff. Yet it is often seen as a poor description of actual behavior because it does not exploit possible deviations by the opponent, but instead focuses on guaranteeing a minimum outcome.

A more general characterization of rational choice would therefore be desirable, one not necessarily tied to a "negative" criterion like Minimax. John Nash famously supplied such a solution: the "Nash Equilibrium." (Nash 2002a)[20] He also showed that in 2PZSG the Minimax and the Nash equilibrium coincide. Yet the Nash Equilibrium, the central solution concept in much of contemporary economics, is less intuitive as a prescriptive account of rational behavior than von Neumann's. This points to a persistent substantive tension in Game Theory and Rational Choice Theory: if treated as prescriptive, they may not be predictive, because the prescriptions can be difficult to follow in real interaction; if treated as predictive, they are hard to use prescriptively, because it is rarely possible to establish with certainty whether a real-world situation corresponds to a Nash Equilibrium.[21]

A further issue concerns the taxonomy of games. Indeed, the foundational distinction between "Cooperative Games" and "Non-cooperative" games also shapes competing views of rational behavior. Even if rational choice is comparatively intuitive in cases of pure opposition and a small number of players, it becomes more complex when the number of players increases, the rules permit binding agreements, or the temporal horizon of the game is ill-defined.

In *Theory of Games and Economic Behavior*, von Neumann and Morgenstern defined a game as the "totality of the rules which describe it" (Neumann and Morgenstern 1944, p. 48). Among these rules is the degree of cooperation permitted among players. Yet they did not explicitly addressed this point. In their framework, every n-player game is solved through the formation of coalitions, and they propose a specific solution concept, the "stable set" (see below). The possibility of forming coalitions thus yields a distinct approach to solution.

20 This idea refers to strategic or "non-cooperative" games . Each player i in the game has a set of strategies S, from which he selects some $s_i \in S_i$. If this selected strategy is the best response to the other players' strategies for all the players in the game, then this is a Nash Equilibrium. Assume a strategy s_i and a strategy s_{-i} (which denotes the strategy selection for all players but i). A Nash equilibrium is written as $u_i(s_i^*, s_{-i}^*) \geq u_i(s_i, s_{-i}^*)$ for all players).

21 In other words, it is intuitive why a Nash Equilibrium is the solution of a strategic game, as well as why the minimax is the solution to a 2PZSG. Less intuitive and more debatable is why a Nash Equilibrium is played. The idea of "rationalizability" was introduced to cope with this problem. This notion entails the immediate idea that each player could be rational even if her beliefs are incorrect. Namely, a strategy could be the best response to another player's move, which is compatible with the idea that the other player is choosing her best response to player one's strategy, and so on. Nash Equilibrium is then a rationalizable strategy, but not all rationalizable strategies are Nash Equilibria. Douglas Bernheim and David Pearce first introduced this notion separately in 1984 (see Bernheim 1984; Pearce 1984)

The explicit distinction between "Cooperative" and "Non-cooperative" games was introduced by Nash, with reference to the degree of communication permitted among players (Nash 2002b; Nash 2002a). In settings where communication is impossible, each player chooses the course of action she believes will yield the highest payoff (a case that resembles how economists traditionally modeled competitive markets). Nash established the fundamental theorems for such games, most notably the "Nash Equilibrium," i.e., a strategy profile in which no player has an incentive to deviate given the strategies chosen by the others.[22] Although the Nash Equilibrium is the most influential result in Game Theory, it applies to "non-cooperative" settings; and Nash's original framework required subsequent refinements to handle richer environments, such as multistage interaction or imperfect information—refinements that, in some cases, appeared only decades later.

In 1953, Nash returned to this distinction in his discussion of bargaining theory. He analyzed a setting in which two agents have opposed interests (although "neither completely opposite nor completely coincident," Nash 2002b, p. 99) and an incentive to reach an agreement through negotiation. This setting is "cooperative" because "the two individuals are supposed to be able to discuss the situation and agree on a rational joint plan of action, an agreement that should be assumed to be enforceable." (ibidem)[23] After Nash, von Neumann and Morgenstern's framework came to be labeled "cooperative."

In *The Theory of Games and Economic Behavior*, the authors had already introduced several notions that later became standard in Game Theory, such as matrices for representing two-person games and game trees for representing multistage interaction (respectively, games in "normal form" and in "extensive form").[24] They also showed how games can be represented through set-theoretic notions, such as partitions, to address the degree of information among players; and they offered a precise characterization of "strategy" as a function that associates each possible course of action with an expected payoff (Neumann and Morgenstern 1944, 79 et ss.). Yet their most detailed analysis concerns games that allow coalition formation and whose solution is therefore represented by a set of admissible arrangements, rather than, as in Nash's later framework, by a strategy profile. This highlights the difference between solving a "cooperative game" and identifying a Nash equilibrium in a non-cooperative game.[25]

22 Not all non-cooperative games have a Nash Equilibrium. To provide the existence of a Nash Equilibrium, the set of actions of each player must be non-empty, compact, and convex, and the preference relation on this set must continuous and quasi-concave (to explore this point, see any intermediate textbook of Game Theory, for instance, Osborne and Rubinstein 1994).

23 See also how Duncan Luce and Howard Raiffa, in their crucial exposition of Theory of Games, published in 1957 (see below) defined "Cooperative games": "By a *cooperative game* is meant a game in which the players have complete freedom of pre-play communication to make joint *binding* agreements. In a *non-cooperative game* absolutely no pre-play communications are permitted between the players." (Luce and Raiffa 1957, p. 89 Italics in the text)

24 The authors used the expression "normalized form." Neumann and Morgenstern 1944, p. 85.

However, it should also be noted that the distinction between "cooperative" and "non-cooperative" games has, to some extent, been softened, beginning with Nash himself. In his last decisive contribution to Game Theory, he attempted to provide "non-cooperative" foundations for "cooperative games." (Nash 2002c) [26] He thus developed a bargaining model in which the bargaining solution is derived from a non-cooperative threat game. This initiated a research program, the "Nash Program", that expanded especially after the Eighties, in part through pivotal refinements of Nash equilibrium by John Harsanyi and Reinhardt Selten (Serrano 2005; Binmore and Dasgupta 1987).

Finally, another crucial distinction is about the number of players. This is important especially for "cooperative games" because each player's strategic reasoning includes the evaluation of joining alternative coalitions. In such cases, rational behavior can mean very different things: interaction with a single opponent or interaction with multiple players, where a wider set of options is available. For a 2PG, the distinction between "cooperative" and "non-cooperative" settings can sometimes be blurred. In n-person games, by contrast, it is fundamental: it changes solution concepts, as well as the meaning and potential applications of the theory. Indeed, when discussing three-person zero-sum games, von Neumann and Morgenstern wrote:

> "We saw that the zero-sum one-person game was characterized by the emergence of a maximum problem and the zero-sum two-person game by the clear cut opposition of interest which could no longer be described as a maximum problem. And just as the transition from the one-person to the zero-sum two-person game removed the pure maximum character of the problem, so the passage from the zero-sum two-person game to the zero-sum three-person game *obliterates* the pure opposition of interest." (Neumann and Morgenstern 1944, p. 220. My italics)

Whereas "pure opposition of interests characterizes 2PZSG," an n-person game involves the possibility of allying with other players; in other

25 Note also that there are different definitions and domains of Cooperative Game Theory. As the game theorist and mathematician William F. Lucas put it: "[...] in the cooperative case, one assumes that the participants can communicate, form coalitions, and make binding agreements. These games are primarily concerned with which coalitions will form and how the resulting gains (or losses) will be allocated among the participants." (Lucas 1994, p. 544) Besides, game theorist Roberto Serrano also identifies four other definitions in the literature, based on such concepts as "fairness," "enforcement authority," 'normativity' other than coalitions. (Serrano 2005)

26 Nash devoted the remnant part of the Fifties to work on problems of pure mathematics before his dramatic and well-known collapse into mental illness, which inhibited him from scientific activity for almost twenty years. He recovered only in the Eighties and in 1994 was the first game theorist, together with John Harsanyi and Reinhardt Selten, awarded the Nobel Prize in Economics. For a compelling biography of Nash, see Nasar 1998. For a collection of Nash's papers and a review of his scholarly accomplishments, see Kuhn and Nasar 2002

words, a "parallelism of interests" can arise. According to the authors, this makes cooperation desirable and increases the scope for agreement. As they conclude: "Of all this, there can be no vestige in the zero-sum-two-person game. Between two players, where neither can win except (precisely) the other's loss, agreements or understanding are pointless." (Neumann and Morgenstern 1944, p. 221)

This also affects the meaning of rational behavior. Here the dilemma concerns not only the "positive vs. normative" status of rational choice, but also the substantive content of what players do. Under some solution concepts—especially when they yield sets with many elements—asking "what does it mean to behave rationally" risks becoming a question with an indeterminate answer.

2.1.2 The general solution for n-person abstract games

Von Neumann and Morgenstern did not envisage pure opposition of interests when the number of players is greater than two. Consequently, finding a solution to such a game implies identifying a set of players' payoffs that displays some form of "stability."

Since the authors defined a game as the "set of rules which describe it," a solution is the set of rules for each participant specifying how to behave in every situation that may arise. More specifically, a solution can be described as a "set of imputations" (which, in the case of 2PZSG, includes only one element). By "imputation," they meant the outcome for each participant, namely the behavior satisfying reasonable requirements for "optimum behavior." Consider, for example, an n-person zero-sum game: players can combine to exclude the remaining ones; a solution must therefore account for the gains of each player within a coalition, while also recognizing that coalitions may break and that players can join new ones if they can secure higher yields.

Unlike the Minimax discussed above, a solution for the n-person zero-sum game also embodies "an absolute state of equilibrium in which the quantitative share of every participant can be precisely determined."(Neumann and Morgenstern 1944, p. 34) In Morgenstern's words: "In conceiving of the general problem, a social economy or equivalently a game of n-participants, we shall expect the same thing: a solution should be a system of imputations possessing in its entirety some kind of balance and stability the nature of which we shall try to determine."(Neumann and Morgenstern 1944, p. 36)[27]

[27] These words, and the next paragraph refers to the first chapter of *The Theory of Games and Economic Behavior*. This chapter is likely Morgenstern's only substantive contribution to the book. Many of the complex mathematical techniques von Neumann employed were out of reach for Morgenstern. Indeed he, as customary at the time, lacked advanced training in mathematics (even if through his friendship with Viennese scholars, like Karl Menger and Abraham Wald, other than von Neumann himself, Morgenstern's understanding of mathematical economics was by far superior to many economists of his time, like for instance Schumpeter, or Hayek). However, Morgenstern's contribution cannot be downplayed. The first chapter indeed links the theory of games with the current debates in economics, of which von Neumann was not entirely aware. Besides, this chapter is important also because

Generally speaking, a solution V is a set of payoff distributions (or "imputations") x and y such that the following conditions hold:

1. no $x \in V$ is dominated by an $y \in V$
2. every $y \notin V$ is dominated by an $x \in V$

In an n-person game, different arrangements of players are possible, and payoffs can be distributed in different ways. A solution is a set of "distributions" that displays both "internal stability" and "external stability."[28]

Morgenstern explicitly related the notion of imputation to that of partial equilibrium in economic theory. He went further, however, in the case of "sets of imputations," linking them to standard patterns of behavior in social organizations. In a social context, the authors argued, the system of imputations describes the "established order of the society." Moreover, since the set of imputations also has internal stability, "once they are generally accepted, they overrule everything else and no part of them can be overruled within the limits of the accepted standards." (Neumann and Morgenstern 1944, p. 42) A potential weakness of this notion lies in the lack of "uniqueness," i.e., the fact that many (potentially infinite) solutions to a single game can exist.[29] Nevertheless, according to the authors, this is not necessarily a problem. If stability refers to a "standard of behavior," then "given the same physical background, different "established order of society" or "accepted standard of behavior" can be built, all possessing those characteristics of inner stability which we have discussed." (ibidem)

The largely verbal character of these pages, together with the analogy between a game's solution and different "standards of behavior," led some scholars to speak of an "institutionalist side" of von Neumann and Morgenstern's thought (Giocoli 2003b; Schotter 1992). This is unsurprising. The authors stated that "the procedure of the mathematical theory of games of strategy gains definitely in plausibility by the correspondence which exists between its concepts and those of social organizations." (Neumann and Morgenstern 1944, p. 43) They thus suggested that Game Theory could offer an alternative framework for the social sciences, where "almost every statement which we - or for that matter, anyone else - ever made concerning social organizations runs afoul of some existing opinion. And by the very

the general significance of the game theory concepts is explained in verbal terms, easy to follow also for the mathematically untrained reader (on Morgenstern's role in creating the Theory of Games see: Morgenstern 1976b; Schotter 1992; Rellstab 1992; Leonard 2010).

28 "Internal stability" and "external stability" are habitual definitions nowadays for referring to the conditions above (see Shubik 1984) However, neither von Neumann and Morgenstern nor early textbooks, like Luce and Raiffa's, adopted them. "Domination" refers to the fact that imputations are vectors. Therefore it does not simply mirror the idea of "greater" or "lesser." For example, x dominates y when a group of participants prefers x to y and can form a coalition, an "effective set" for x over y. If such a set exists for y over z, it does not logically imply an "effective set" for x over z.

29 But to fully grasp this aspect, the verbal definition is not sufficient anymore, and the mathematical treatment is necessary

nature of things, most opinions thus far could hardly have been proved or disproved within the field of social theory." (ibidem) This also helps explain why many scholars—including von Neumann and Morgenstern—took for granted that game theory's natural boundaries extended to the social sciences in general, not only to economics.[30]

The discussion presented above is developed rigorously in the sixth chapter of the 1944 work (Neumann and Morgenstern 1944, pp. 238–90). Here, the reader is guided through a quasi-textbook, step-by-step procedure for defining coalition concepts formally, namely, each coalition's payoff and the distribution of that payoff in a way that makes the outcome acceptable to each player.

The first step is to reduce the complex possible outcomes of a game to a single number. In mathematical terms, the device adopted is a real-valued function defined on the set of all subsets of $N = 1, ..., n$ (the set of all players), i.e., the set of all possible coalitions. This function, called the "characteristic function" and denoted $v(S)$ for each $S \subset N$, assigns a numerical value to each coalition. To determine the characteristic function, von Neumann treats each n-person game as a 2-person game, where players 1 and 2 are, respectively, the coalition and its complement. One can then apply the minimax theorem and determine a single value for each coalition. (Neumann and Morgenstern 1944, pp. 239–40)

The characteristic function captures "everything that can be said about coalitions between players, compensations between partners in every coalition, mergers or fights between coalitions." (Neumann and Morgenstern 1944, p. 240) At the same time, it does not specify how to divide the coalition value among its members. From a mathematical point of view, $v(S)$ is not simply the sum of individual payoffs within the coalition. Rather, it is defined so as to account for the options each player has to leave one coalition and join another. Mathematically, $v(S)$ has three main properties:

$$v(0) = \emptyset$$

$$v(S) = -v(S)$$

$$v(S \cup T) \geq v(S) + v(T) \text{ if } S \cap T = \emptyset$$

Intuitively, the first means that the value of a coalition without players is zero. The second indicates that the value of a coalition is the negative of its complement (by reduction of the n-person game to a 2-person game and the zero-sum condition). The third means that the value of a coalition formed by the union of two disjoint coalitions cannot be less than the sum of their values. From these properties, others follow logically:

$$v(N) = 0$$

30 Take, for instance, the foreword and the collection of essays edited by economist Martin Shubik (who was a student of Morgenstern at Princeton), under the title *Readings in game theory and Political Behavior.* (Shubik 1954)

$$v(S_1 \cup \cdots \cup S_n) \geq v(S_1) + ... + v(S_n)$$

if $S_1 ... S_n$ are pairwise disjoint subsets of N. And

$$v(S_1) + ... + v(S_n) \leq 0$$

if $S_1, ..., S_n$ are pairwise disjoint subsets of N with the sum N.

The core of von Neumann's argument is that the analysis of any n-person zero-sum game can be carried out by means of characteristic functions.[31] To show this, he introduces the concept of "strategic equivalence." If in two games Γ and Γ', the strategic possibilities are the same and the only differences consist in fixed payments to each player,[32] then the two games are equivalent. He also introduces the notion of the "reduced form" of $v(N)$, i.e., $\bar{v}(S)$.[33]

A further aspect is the distinction between "inessential" and "essential" games.[34] If $\bar{v}(S) = 0$ for all S, the game is "inessential": no coalition generates a surplus, because each player can obtain alone what he can obtain in coalition with others. The opposite case, where $\bar{v}(S) > 0$, defines an "essential" game. Only essential games are of interest for von Neumann and Morgenstern's analysis.[35]

Finally, another key point concerns "symmetry" and, relatedly, "fairness." Symmetry was introduced in the discussion of 2PZSG and means that switching the players' roles does not change the game. Beyond the implications this has for how one conceptualizes players' individuality, symmetry becomes even more important in n-person games.[36]

31 Von Neumann also provided elementary proof of the properties above (Neumann and Morgenstern 1944, pp. 241–3. Very intuitively: if the value of the coalition without players is zero, and the game is zero-sum, its opposite, namely, the coalition of all players has a zero value too. If each coalition cannot be valued less than the sum of its pairwise disjoint subsets, this holds for any number of subsets. Finally, from above, if the sum of pairwise disjoint subsets is N, then the value of the coalition of their sum cannot be more than zero.

32 In formal terms: $v'(S) = v(S) + \sum_{k \subset s} a_k$ where $a_1, ..., a_k$ represents what plays k obtains in Γ' more than in Γ

33 Practically, what von Neumann is doing is simply providing a method for treating the characteristic functions numerically in a given game. Therefore, by normalising the values of one-element sets as γ and the values of every $(n-1)$ elements set as $-\gamma$, it is possible to obtain the value of each p-element set: this can be written as: $-p\gamma \leq \bar{v}(S) \leq (n-p)\gamma$ (see also: Luce and Raiffa 1957, pp. 185–9). The role of the inequalities above is further explored in Neumann and Morgenstern 1944, pp. 252–3. Note that for $n = 3$, it has a definite value, $0, 1, n-1, n$. This inequality represents the "range" of possible values for each normalized Characteristic Function for every number of elements in S.

34 In reality, this distinction has been yet introduced in the previous chapter, discussing the case of $n = 3$, but now the discourse is generalized to $n > 3$, and using the 'reduced form' of the game.

35 Another way of defining 'inessentiality' in terms of the properties of characteristic function is the following: $v(S \cup T) = v(S) + v(T)$ (Neumann and Morgenstern 1944, p. 251). This is called "additivity". Therefore, essential games have a non-additive characteristic function.

36 The condition of symmetry has been so defined: "the symmetry [...] requires that

Having established the axiomatic foundations of the characteristic function, von Neumann derives solution concepts for n-person games, beginning with the simplest case of a 3-person zero-sum game (Neumann and Morgenstern 1944, pp. 260–3). In this setting, the structure of the game is determined by two-player coalitions. Assuming the reduced form ($\gamma = 1$), the following characteristic functions for different coalitions are possible:

- $v(0) = 0$
- $v(1) = -1$
- $v(2) = 1$
- $v(3) = 0$[37]

Considering two-player coalitions, these are $\{1,2\}, \{2,3\}, \{1,3\}$. Accordingly, for each coalition the following payoff distributions are possible: $v(1,2) = (\frac{1}{2}, \frac{1}{2}, -1)$, $v(1,3) = (\frac{1}{2}, -1, \frac{1}{2})$, and $v(2,3) = (-1, \frac{1}{2}, \frac{1}{2})$. Each member of the coalition splits the total value, while the excluded player receives -1 (the value of one-member coalitions). These distributions, labeled "imputations," exhaust the game's strategic possibilities. The solution is the set of all three. Only this set yields a stable equilibrium, but stability is a property of the three distributions taken together. Indeed, each coalition can be circumvented, for instance, if one player defects and forms a new two-member alliance with the previously excluded player.[38]

In the general case of the n-person zero-sum game (Neumann and Morgenstern 1944, pp. 263–72), an imputation has the following formal properties:

$$a_i \geq v(i) \quad \text{for } i = 1 \ldots n$$

and

$$\sum_{i=1}^{n} a_i = 0$$

These properties correspond to the intuitive ideas of "individual rationality" and "Pareto optimality" (even if the authors did not label them in this way in the 1944 work). The first states that a player, whether inside a

the names of the players play no role in determining the value, which should be sensitive only to how the characteristic function responds to the presence of a player in a coalition. In particular, the symmetry axiom requires that players who are treated identically by the characteristic function be treated identically by the value." (Roth 1988, p. 5)

37 Where the number in brackets indicates the number of players in each coalition.

38 In von Neumann and Morgenstern's words: "In each of the three distributions [...] there is, to be sure, one player who is desirous of improving his standing, but since there is only one, he is not able to do so. Neither of his two possible partners gains anything by forsaking his present ally and joining the dissatisfied player: already gets $\frac{1}{2}$, and they can get no more in any alternative distribution."(Neumann and Morgenstern 1944, pp. 262–3)

coalition or alone, should not accept a payment below what he can secure by acting alone. The second implies that the sum of imputations cannot exceed the value of the grand coalition (which, as seen, is zero). At the same time, it cannot be less than zero, because in that case some player could gain without any loss to the others (Luce and Raiffa 1957, pp. 192–3). Mathematically, an "imputation" is a vector in an n-dimensional vector space L_n. A subset $S \subseteq N$ is 'effective' if:

$$\sum_{i \in S} a_i \leq v(S)$$

An imputation $\vec{a}$ dominates another imputation $\vec{b}$ (i.e., $\vec{a} \succ \vec{b}$) if there exists a set S with the following properties::

$$S \neq \emptyset$$

$$S \quad \text{is effective for } \vec{a}$$

$$a_i > b_i \quad \text{for all } i \in S$$

A set of imputations, V, is a solution if it satisfies the following properties:

- no $\vec{b} \in V$ is dominated by a $\vec{a} \in V$
- $\forall \vec{b} \notin V$ is dominated by some $\vec{a} \in b$ [39].

The elements in V are precisely those not dominated by any element of the set, and which dominate all imputations outside the set. This is the same definition given above, but now each term has a precise formal meaning.

Yet, although these definitions clarify how the solution of an n-person game can be interpreted as a "standard of behavior" and can display "some kind of stability," the same game rarely has a unique solution. Hence, "several stable standards of behavior may exist for the same factual situation. Each of these would, of course, be stable and consistent in itself, but in conflict with all others." (Neumann and Morgenstern 1944, p. 266) The authors show, for instance, that in an inessential game there exists precisely one imputation, namely the vector such that $a_i = v(i)$. In an essential game, by contrast, infinitely many imputations exist, though not the one above.[40]

[39] Note that this does not exclude the existence of some $\vec{b} \notin V$ which dominates an $\vec{a} \in v$

[40] Despite the infinitely many possible solutions, von Neumann conjectured that V is never empty (Neumann and Morgenstern 1944, pp. 277–8). William Lucas proved that this is not true, and the V-set for certain games may be empty (Lucas 1994).

2.1.3 The reception of von Neumann and Morgenstern's Theory of Games

Works as the aforementioned by Giocoli and Leonard explored the reception of *Theory of Games* among economists. To summarize developments after the publication of von Neumann and Morgenstern'swork: scholars interested in mathematical economics widely recognized it as outstanding, but only some of its components drew sustained attention. These include, as seen, the Expected Utility Theory and von Neumann's introductory discussions of topological ideas such as convexity and linearity, and their use in solving optimization problems.

The trajectory of the stable set solution illustrates this pattern. From a general standpoint, this concept was quickly supplanted by the distinct (and conceptually different) notion of Nash Equilibrium. Even within the domain Nash labeled as "cooperative", mathematicians and game theorists developed alternative solution concepts in the Fifties to address those features of social situations that were problematic in von Neumann and Morgenstern's framework.[41]

Indeed, as shown by empirical tests conducted at RAND in 1952, it was not easy to assess the von Neumann–Morgenstern solution because it was not entirely clear what the theory asserted (Leonard 2010, p. 328; Kalisch et al. 1952). Mathematicians and game theorists thus recognized that the solution concept was too broad to yield an adequate predictive or descriptive account of real-world situations.

Von Neumann and Morgenstern treated the possible existence of infinitely many solutions as reflecting the fact that a rational player may face multiple equally adequate courses of action. To avoid this conclusion, which would undermine practical applications of Game Theory, many alternative solution concepts for the same kinds of social situations were proposed. Some, such as the "Core" or the "Shapley Value," were readily extended to political analysis. Yet none seemed decisively superior to von Neumann and Morgenstern's solution, at least in Riker's eyes. Accordingly, he grounded his analysis of political coalitions in a specific subset of their framework (Riker 1962b).

Theory of Games nonetheless attracted a remarkable set of reviewers, including Herbert Simon Leonid Hurwicz (future Nobelists), Jacob Marschak, the statistician Abraham Wald, and mathematicians such as

[41] Duncan Luce and Howard Raiffa's *Games and Decisions* represents the first comprehensive analysis of early game-theoretical contributions up to the mid-Fifties and the work many students and scholars used in their training in Game Theory tools. It is also the case with Riker. In their textbooks, the authors provided at least 5 different solutions ideas for n-person games, other than von Neumann and Morgenstern's. These are the "core," the Ψ-stability, the "reasonable outcomes," and the "Shapley-value" (Luce and Raiffa 1957, pp. 180–252). Furthermore, in a way that would become customary in the analytical treatment of the topic, they started with the "core," which, although successive to the "stable set," generalizes it. Note, finally, that the number of solution ideas for such games continued to grow. For instance, in a "middle-level" review by Martin Shubik, in the Eighties, the list presented totaled 8 main concept, which partially overlapped that provided by Luce and Raiffa (Shubik 1984).

Arthur Copeland.

Simon's review appeared in the *American Journal of Sociology*, while Copeland's was published in the *Bulletin of the American Mathematical Society.* Both came out in 1945, together with the perhaps most famous review by Hurwicz in *The American Economic Review.* In 1946 and 1947, Jacob Marschak at the Cowles Commission[42] and Abraham Wald published their reviews, the latter partially reprinted in Shubik's collection (Simon 1945; Copeland 1945; Hurwicz 1945; Marschak 1946; Wald 1947; Shubik 1954). Marschak's and Wald's reviews played an important role in the early diffusion of von Neumann and Morgenstern's work among economists, even if the initial enthusiasm among mathematical economists soon gave way to other developments, most notably general equilibrium.[43]

Simon's review had less impact. Yet its perspective is particularly interesting because Simon approached this work as a social scientist rather than as an economist. Copeland's review, by contrast, largely restates the essential points of the theory. Still, Copeland's role in disseminating *Theory of Games* within the mathematical community beyond Princeton, it will be shown, is significant and often neglected.

Herbert Simon is well known among social scientists: he coined the notion of "bounded rationality" and was awarded the Nobel Prize in Economics in 1978. Less often noted is that he was trained neither as an economist nor as a mathematician. He was a political scientist educated in Merriam's "Chicago School," although his research interests centered on policy-making and organization theory (Simon 1996). Simon begins his review by explicitly recognizing the importance of this work for the social sciences as a whole. As he wrote: "[a]lthough no explicit applications are made to sociology or political science, the schema is of such generality and breadth that it can undoubtedly make contributions of the most fundamental nature to those fields." (Simon 1945, p. 637) He acknowledged the rise of mathematical economics while also noting that no comparable effort existed in other social sciences, aside from the work of figures such as Talcott Parsons or the quantitative sociologist Stuart Dodd, whose formalism was not mathematical. He continued: "The *Theory of Games* is both more modest and infinitely more impressive than any of these earlier attempts. It seeks merely to develop in systematic and rigorous manner a theory of rational human behavior." (Simon 1945, p. 639) Simon also emphasized the mathematical novelty of von Neumann's approach, namely his use of set theory and topology rather than calculus and differential equations.

In Simon's view, the second chapter, where the authors describe both informally and formally the notion of "game", contains the most important contribution of von Neumann and Morgenstern's work to the social sciences.

> "Sociology has been forced to treat of human behavior (at least in its rational aspects) in terms of "ends" and "means"; for example, these are fundamental categories in *The Structure*

[42] See the first chapter

[43] However, Marschak's championing of expected utility had a substantial impact on his reception among economists. (See Moscati 2018)

> *of Social Action*. It could easily be shown that these two terms complicate rather than simplify the analysis of human rationality, and it is to be hoped that they will now be discarded, both in sociology and in ethics, in favor of the schema of "alternatives," "consequences," and "values" attached to "consequences" (the terminology here is the reviewer's and not that of *Theory of Games* which the description of games of strategy provides. This schema quite obviously owes its origins to the utility calculus of economics, but in its generality it can be applied, at least descriptively, to all behavior, whether rational or not." (Simon 1945, pp. 638–9)

Within this scheme, it becomes possible, for the first time, "to define unambiguously and to analyze the concepts of 'competition' and 'cooperation' which have become such important categories of sociological political and economic theory." (ibidem) Simon also pointed to its relevance for a theory of administrative behavior. Turning to the general solution of n-person games, Simon noted that "the concept of 'stability' [...] is perhaps not entirely free from objections in its details" but that "it certainly points in a proper direction," while also offering a precise account of coalition formation (ibidem).[44]

Simon went further, providing a list of sociological and political topics that could potentially be addressed using game theory: "For example, it should be possible to identify the theory of revolutions with the theory of stability and instability of "standards of behavior" in certain games. For this purpose, the theory will probably have to be developed from a static to a dynamic one, however. In the field of politics, one might construct games which would illustrate the formation of two-party or multi-party systems, respectively, and this could lead to a comparison of the circumstances favoring one or another type of equilibrium." (ibidem)

Unlike Simon, Arthur Copeland was a mathematician rather than a social scientist. He therefore offered a long (16-page) and relatively detailed exposition of the *Theory of Games* (Copeland 1945). Copeland's primary research field was probability theory. Writing for a mathematical audience, he limited himself largely to describing the content of the work, including some notation. Unlike Simon, he did not propose specific applications to social-scientific problems.

Copeland nevertheless had a particular interest in voting theory. He proposed a pairwise voting method later labeled after him.[45] More importantly for the subsequent dissemination of Game Theory, Copeland was Howard Raiffa's Ph.D. advisor at the University of Michigan. Raiffa, as seen, later co-authored with Duncan Luce the most influential Game Theory textbook prior to the boom of Game Theory in economics, *Games*

44 However, in a personal exchange he had with Morgenstern, Simon was more critical on this point. (Leonard 2010, pp. 260–1)

45 To sum up, each candidate obtained a point for each pairwise comparison he won (and half-point if there were a tie). The election was won candidate with the greater number of points

and Decisions. Raiffa recalled that Copeland chaired a small seminar in 1948-9 and 1949-50, devoted to game theory, with particular attention to 2PZSG and their extensive form (Raiffa 1992). Copeland's influence on the development of Game Theory was therefore indirect, but not negligible.

However, when Shubik was asked to edit a collection of essays intended to show possible extensions of game theory to political behavior, he included neither Copeland's comprehensive review nor Simon's suggestive remarks. He opted instead for Wald's equally valuable review, and complemented strictly game-theoretic readings with related theoretical work not strictly in Game Theory, such as Arrow's analysis of social choice and Duncan Black's essay on the unity of economics and political science.

2.2 Formal Theories of Politics in the Fifties

Game theory gave an important impulse to the development of mathematical economics, although the results were, as seen, quite different from the authors' original expectations. Parallel to this scientific development, formal approaches were also extended to the political and social sciences during the Fifties.[46] The fields involved included the study of collective choice and voting behavior, as well as long-debated issues such as "power."

The Fifties opened with the publication of Kenneth Arrow's innovative analysis of Social Choice, *Individual Values and Social Choices*, but already in the late Forties the Scottish economist Duncan Black had addressed similar problems. In 1954, the Princeton and RAND-based game theorist Lloyd Shapley,together with Martin Shubik, published a brief paper in the *American Political Science Review* in which, drawing on Shapley's general solution for n-person games, they proposed an innovative formal analysis of power. Finally, the study of international politics increasingly relied on game-theoretic techniques, aiming at greater analytical precision in the treatment of Cold War issues.

Black's work and that of Anthony Downs became foundational for the field of spatial analysis of voting and elections. In the same period, James M. Buchanan and Gordon Tullock laid the groundwork for what would become known as the "Public Choice" approach in economics (Arrow 1951b; Black 1958; Downs 1957; Buchanan and Tullock 1962).

As shown above, these formal theories did not become mainstream within political science during the Fifties. Only after Riker's intellectual commitment and his appointment at the University of Rochester did the formal approach, what he labeled "Positive Political Theory", emerge as a clearly defined subfield of the discipline. Moreover, there were important differences among the early attempts to develop formal political analysis. The most relevant concerned the role of Game Theory. As will be shown in the next chapter, Riker encountered Game Theory in the second half of the Fifties and became deeply committed to it, adopting at times a distinctly "lobbyist" stance toward its application to political analysis. By contrast, the works of Arrow, Black, and Downs did not employ Game

46 A few words defining the meaning of "formal" will be spent in the next section

Theory directly, despite drawing on some closely related ideas. The case of Shapley and Shubik is again different and will be discussed in relation to Riker in the following chapter.

2.2.1 Economists and Political Scientists

As economists' language became increasingly mathematical, some scholars came to regard this as the natural, or at least the most effective, language for the social sciences as a whole. With varying degrees of awareness of the long intellectual traditions that preceded them (especially in Arrow's case), they revisited longstanding political questions using the powerful tools of mathematical formalism. At the same time, although the "Behavioral Revolution" profoundly shaped the environment of American political science, it never assumed a formal character. This divergence widened the gap between economics and neighboring disciplines. As a result, contributions made by economists were not easily absorbed into political science during the Fifties.

It is difficult to specify what it means "to be formal" in disciplines other than mathematics. As a large body of scholarship has shown, the development of mathematical economics was characterized by the adoption of a well-defined set of techniques and tools. As discussed earlier, this process was reinforced by explicit connections to a particular conception of mathematics, namely the "formalist program" and its variants (such as Bourbakism). At the same time, however, this path, as well as its outcome, was far from inevitable or unique (Weintraub 2002 and Ingrao and Israel 1987).

In the present discussion, a broader definition of "formal" is adopted. By "formal political theory", it is meant a kind of political theory grounded in individual action, choices, and preferences, that is, broadly speaking, in economic theory. In a letter written in the early Sixties, Riker defined "formal" as meaning that a theory can be expressed in algebraic rather than purely verbal terms ("Supplementary Statements", Riker to Tyler, December 4th, 1959: Riker 1959b; Riker n.d.). This definition also implies that reliance on mathematical and logical proofs is a necessary condition for a well-formulated emerging theory.

A consequence of this definition is that, among the authors mentioned above, only Arrow's Social Choice Theory fully qualifies as formal in this sense. Black's use of mathematical language was less sophisticated than Arrow's, even though he rigorously proved some of his results, most notably the "Median Voter Theorem." Downs's work, by contrast, was almost entirely verbal, despite his training as an economist.

These works were produced within economics by scholars trained as economists (with Arrow representing the cutting edge of mathematical economics).[47] At the same time, these authors were well aware of the existence of an established research tradition in political science. Consequently, they often justified their use of formal analysis through methodological

[47] The only exception is Gordon Tullock, who was trained as a legal scholar

arguments, or by stressing the close affinity between traditional political problems, such as voting, and collective choice issues long studied by economists, particularly in welfare economics.

To address some terminological issues: what has been described above as formal theory was labeled by Riker, already in the late Fifties, as "Positive Political Theory," or "Formal, Positive Political Theory" (Riker 1962b, p. 33). Formalism was its defining feature. Yet authors such as Black, Buchanan, Tullock, and Riker himself never provided a precise definition of what "formal" meant, often implying that it referred to "what economists do." Before the label "Positive Political Theory" became widespread in American political science (and "Public Choice" in economics), other terms were employed. Black spoke of a "Pure Science of Politics" (Black 1950; Black 1958); Tullock referred to a "Strict Theory of Politics" (Buchanan and Tullock 1962); and Riker invoked the idea of a "Genuine Science of Politics" (Riker 1962b). All these expressions indicate a shared ambition to make political science genuinely scientific. Economics, by then the only social science in which formal modeling was widely established, naturally appeared as a model to emulate. However, this relationship with economics proves more problematic, especially with regard to the connection between formal theory and empirical validation in Riker's work.

Political scientists did not begin to look to economics as a model only in the Fifties. One may recall, for instance, Graham Wallas at the London School of Economics (Wallas 1920). Similarly, Charles Merriam, in comparing the methods of political science with those of other disciplines, referred explicitly to economic analysis. In a 1923 article in the *American Political Science Review* on "recent advances in political methods," Merriam,like Wallas, described economics as moving away from a priori reasoning (characteristic of classical political economy) toward the progressive integration of statistical analysis and psychological insights (Merriam 1923). For Wallas and Merriam, however, undoubtedly both central figures in the early twentieth-century push toward a more scientific political science, economics was not to be followed into the realm of "high theory," but rather in the more concrete domain of quantitative and empirical analysis. Moreover, they encouraged political scientists to look not only to economics but also to disciplines such as biology and anthropology, as well as historical and sociological studies. Finally, both emphasized psychological insights in economic models of choice, presenting a somewhat simplified account of the development of economics that would later be echoed in the behavioral revolution.

The dramatic growth of formal modeling in economics from the late Thirties onward made it possible to reframe classical political issues in mathematical terms. The mathematical turn of the Fifties aimed to define political problems with greater precision and to provide internally consistent theories of how political systems operate and resolve conflict. The tools developed within economic theory, starting with Game Theory, appeared well suited to this task. Yet the emphasis on formal reasoning tended to downplay psychological and behavioral considerations, replacing them with concerns about logical consistency. This marked a significant departure from the views of Wallas and Merriam and highlights the distinctive character of formal modeling in the Fifties and Sixties.[48]

2.2.2 Social Choice and Voting: selected aspects of Arrow, Black, and Downs

This analysis is not intended to be exhaustive. Rather, the focus is on some methodological features of their models. Arrow and Black, as will be shown, made partial use of ideas derived from the *Theory of Games*. In particular, Arrow' treatment of rational choice closely resembles the axiomatization of utility theory developed by the founders of Game Theory. Black, for his part, also addressed strategic considerations in politics, though his conception of rationality differs markedly from Arrow's.

Although his most influential book appeared in 1958, Black's early results on voting theory date back to the late Forties. For this reason, the discussion begins with him. Duncan Black occupies a central position among scholars who analyzed the relationship between individual choice and collective decision-making. Together with the American economist Anthony Downs, he is commonly regarded as a founder of spatial analysis in political science.[49] The "spatial" aspect refers to the representation of voters' preferences as locations in an issues space, often depicted using Cartesian coordinates. Under these assumptions, Black derived a simple but powerful result: in majority-rule decisions on a single issue, the median voter's preference is decisive. Downs generalized this insight to one-dimensional party competition, where parties converge toward the median voter. These results were later extended and formalized, often using game-theoretic tools, from the Sixties onward.

Black's early results are also connected to Arrow's arguments regarding what he called the "General Possibility Theorem for Social Welfare Functions" (Arrow 1951b). Although voting paradoxes had been identified much earlier (most importantly by French eighteenth-century authors such as Jean Charles de Borda and Condorcet) the works of Arrow and Black marked the emergence of a new subfield: the analysis of collective choice through rational choice methods (Black 1958; McLean 2015).[50]

This parallel notwithstanding, there are significant differences between Arrow's work and Black's, beginning with the scope of their research and the degree of generality and mathematical sophistication. Black was closer

48 Note, however, that these issues were never entirely separated, either in economics or in decision theory. See, for instance, Moscati 2018. As will become apparent in the discussion of Riker's work, this tension also bears on questions of empirical validation and the "positive" aspirations of formal theory.

49 The American statistician and economist Harold Hotelling anticipated some of these ideas (Hotelling 1929). Hotelling directly influenced Downs, whereas Black focused on committee decisions and voting. However, Hotelling's original contribution concerned a different problem, namely spatial competition in a duopoly with homogeneous goods, but he also tentatively advanced political analogies. Notably, Hotelling was Arrow's Ph.D. supervisor at Columbia University (on Hotelling see Gaspard and Muller 2021).

50 Black devoted the entire second part of his 1958 book to reconstructing these debates historically, up to Victorian Britain.

to the idea of a "genuine science of politics," later emphasized by Riker, and can therefore be read as a precursor of both "Positive Political Theory" and "Public Choice." Indeed, in the Sixties he became well acquainted with both intellectual communities. Whereas for Arrow voting was one example among many of collective choice based on individual preferences (namely axiomatized social welfare functions), for Black it was a positive phenomenon to be analyzed through formal theory and, crucially, through empirical inquiry.[51]

Black graduated from the University of Glasgow, where he studied mathematics and physics but developed a strong interest in economics and politics. In the Thirties, at the Dundee School of Economics, he formed a lifelong friendship with the future Nobelist Ronald H. Coase.[52]

Because he was trained in the natural sciences more intensively than many economists of his generation and displayed a pronounced interest in analytical methods for the social sciences, Black aimed to develop a "pure science of politics." This was also the title of an early draft of his major work, later published in 1958 under the less ambitious title *The Theory of Committees and Elections* (Black 1958). The 1958 volume collected papers written in the second half of the Forties and published in leading journals such as *Econometrica*, as well as in the *Journal of Political Economy* and the Italian *Il Giornale degli Economisti*(Black 1948c; Black 1948a; Black 1948b). He also produced a more detailed analytical study with the physicist R. A. Newing, titled *Committee Decisions with Complementary Valuation*. In addition, a long series of papers, many unpublished at the time or published only much later, must be included (McLean, McMillan, and Monroe 1998; Brady and Tullock 1996). Black's early papers were written and published precisely when mathematical economics was taking off within the American economics community. It is therefore unsurprising that they attracted the attention of young scholars, most notably Arrow, who later generalized related results in a different formal framework (Arrow 2014).

Black spent his entire career in the United Kingdom, moving between Belfast, Glasgow, and the University of North Wales at Bangor. He nevertheless served as a visiting professor at several American universities, most notably Chicago, Rochester (after Riker arrived in 1962), and the University of Virginia, where Buchanan and Tullock established the Public Choice School. If he remained somewhat peripheral within British political science, he maintained long and fruitful exchanges with American scholars interested in voting theory.[53]

51 A point also remarked by Arrow: "Black intended his work to be a contribution to the analysis of actual political behavior rather than to that of social welfare. [...]" (Arrow 1951b, p. 79)

52 On this point, see the biographical memoir Coase wrote for Black after the latter's death in 1991 and reprinted in Coase 1994 (on the Dundee School of Economics, established in the Thirties with a vocational orientation, see Tribe 2022).

53 For biographical information about Black, see Coase's preface to McLean, McMillan, and Monroe 1998 and the first section of the editors' introduction in the same volume, as well as Coase 1994. On Buchanan and the Virginia group in the Fifties, see Levy and Peart 2020.

In Black's view, the scope of political science as a discipline is to develop a method for aggregating preference schedules. Accordingly, the starting point of the analysis is the individual, treated as equivalent to a schedule of preferences (in economics, preferences concern goods; in politics, they concern motions among which choices are made). These ideas appear in a methodological paper published in 1950 and are reprised in his early theoretical writings (Black 1950; Black 1958).

In that 1950 article, Black explicitly advanced "the unity of economics and political science" as a necessary step toward building a "pure science of politics." This view rests on the claim that both disciplines are, in effect, subsets of a broader domain, namely the "Theory of Choices." The core of this scientific approach to politics consists in constructing an appropriate set of formal propositions, beginning with a theory of committee decision-making. Black acknowledged that such a theory would not capture the full workings of all committees, but could yield very general propositions for political analysis. In this sense, he wrote:

> "[...] a satisfactory Political Science [...] will have the same distinguishing marks as Walras' *Élements* or Pareto's *Manuel* - or perhaps Marshall's *Principles*, with the admixture of the rigorously formal and the descriptive treatment -rather than those of the existing texts in Politics." (Black 1950, p. 506)

The essential features of this pure theory are, in Black's view, "precisely those of Economic Science" (Black 1950, p. 507), because both disciplines must abstract from empirical complexity in order to handle the social world analytically. What he called "the economic mode of abstraction" proceeds by starting with the simplest problems; only after these are understood can lesser abstraction be introduced. Individual preferences are therefore the starting point of analysis, and these can be represented through preference schedules in the familiar economic style. Although economics concerns situations in which individuals often possess relatively clear information about outcomes (prices and money), Black insisted that "there is no difference in principle between the economic and political estimates which people must make." (Black 1950, p. 511)

Even the central concept of modern economic analysis, equilibrium, can, he argued, be extended to politics. In this context, "political equilibrium" refers to how collective decisions are reached, given individual valuations, through particular voting procedures and institutional adjustments.

The key difference from economics lies in the objects of preference. More specifically:

> "In Political Science the motions before a committee stand in some definite order on the scales of preferences of the members. Equilibrium will be reached through one motion being selected as the decision of the committee by means of voting. The impelling force towards having one particular motion selected will be the degree to which the members' schedules, taken as a group, rank it higher than the others. The barriers to its selection will be of

> two kinds. On the one hand, there is the degree to which the group ranks other motions as high as, or higher than, the motion concerned. And on the other, there is the particular form of committee procedure in use; and it can be shown that with a given group of schedules, one procedure will select one motion, while another procedure will select another.' If so, equilibrium in Politics is 'the resultant of tastes and obstacles '; and these are the words Pareto used of equilibrium in Economics." (Black 1950, pp. 512–3)

As a forerunner of mathematical economics, Black nonetheless appears not to have been closely acquainted with Game Theory at the time. Recognizing that a political theory of this kind would eventually need mathematical formulation, he remarked that much existing mathematics had been developed for physical problems and was not well adapted to the human sciences; therefore, "in time a new Mathematics will be invented" (Black 1950, p. 513, footnote 2). This is a classical, and in some respects surprising, claim, echoing older objections to the mathematization of economics (for instance, the Austrian critique associated with Hans Mayer, well known to Morgenstern). It also parallels the objections addressed by Morgenstern in the first chapter of the *Theory of Games*.

In later works, especially in his 1958 book, Black did gesture toward strategic behavior in politics. In particular, his "Median Voter Theorem" can be framed as a non-cooperative game in which the median preference corresponds to the unique equilibrium outcome. Yet this is not how Black developed his account of strategy. Rather, he focused on parties: parties adapt to institutional environments and may dissimulate their preferences, a point that naturally invites game-theoretic analysis. Black, however, did not formalize this in game-theoretic terms. He was not closely engaged with game-theoretical work and appears to have been skeptical about its ability to yield exact results.[54]

Black's conception of rational behavior further confirms his distance from Game Theory. In the *Theory of Games* there was a visible tension between expected-utility foundations and the notion of rational behavior in n-person games. Still, the axiomatic grounding of utility theory became one of this work's most influential legacies, particularly for Arrow. Black's approach differs. His analysis rests on individual rankings of preferences, but it does not provide a rational choice theory or a formal definition of rationality, either axiomatic (as in Arrow) or as utility maximization. He simply maintained that it would be irrational for an individual to choose an alternative that she does not prefer; hence, if a voter is indifferent between two motions, abstention may be assumed (Black 1958, p. 5).

54 This is suggested by his exchanges with Ronald Coase when Coase was working on the theory of social cost. Black proposed that the relationship between efficient allocation and initial rights assignments could be addressed using game theory, but added that "Neumann and Morgenstern have shown that It is hopeless to attempt anything here in an exact way." (Letter to Coase, July 30, 1959, cit. in Medema, 2020)

Black's views on rationality became explicit in a 1969 paper on Arrow's Impossibility Theorem (Black 1969). There he defended his preference for the term "transitivity" over "rationality," writing:

> "Rational choice' is an emotive term, with the danger that it may induce us to prejudge issues rather than analyse them. In a purely scientific part of a treatment of politics, however, we would wish, so far as possible, to avoid language of this kind and employ only neutral terms. Besides this, the term tends to label alike all procedures which do not secure complete transitivity, whether the intransitivity occurs once in ten cases or once in a hundred million. But, in regard to committee procedures, intransitivity is *essentially a quantitative matter* and 'irrational' would seem to be a wrong designation of a procedure that gave one intransitivity among a hundred million decisions." (Black 1969, pp. 233–4)

As it may be apparent from what said so far, Arrow's work paralleled, and in a certain sense generalized, Black's. Arrow focused on the apparent inconsistency between individual preferences and social choices when the number of possible alternatives exceeds 2 and any admissible ranking of preferences is allowed. As he later recalled, the origins of his interest were twofold: his mathematical training as an economics graduate student at Columbia University under Harold Hotelling, and the stimulating intellectual environments of the Cowles Commission, where he spent time in 1948, and RAND, in 1949 (Arrow 2014). At RAND, scholars such as the philosopher Olaf Helmer conducted game-theoretic analyses with particular attention to international politics. Helmer, as Arrow recalled:

> "[...] was troubled about the application of game theory when the players were interpreted as nations. The meaning of utility of preference for an individual was clear enough, but what was meant by that for a collectivity of individuals? I assured him that economists had thought about the problem in connection with the choice of economic policies and that the appropriate formalism had been developed by Abram Bergson in a paper in 1938; it was a function, called by him the Social Welfare Function, which mapped the vector of utilities of the individual into a utility." (Arrow 2014, pp. 147–8)

Arrow proved that, in an important sense, Helmer's concern was well founded. Bergson had shown how welfare conditions could be derived without summing utilities (and thus without interpersonal comparisons), an approach that contrasted with the earlier Cambridge tradition associated with Marshall and Pigou, and with Pareto's critique. Arrow however, pushed the analysis further and argued that even Bergson's framework was problematic once examined through an axiomatic approach inspired by the mathematician and logician Alfred Tarski.[55] This result, which

Arrow called the "general possibility theorem for Social Welfare Functions," formed the core of his Ph.D. dissertation and was published in 1951.[56]

Although Arrow soon turned to other areas, most notably general equilibrium theory, his elegant argument quickly attracted the interest of young economists, who developed refinements that weakened or modified the conditions yielding inconsistency.[57]

In his 1951 work, Arrow explicitly compared voting and the market mechanism as two forms of collective choice. Both, in this view, are special cases of a broader category of social choice procedures. This parallel notwithstanding, and despite references to political science literature, his work did not belong to political science. Its scope was intentionally general, and its starting point was a critique of welfare economics as a coherent toolset for social policy (Igersheim 2019). Arrow was not primarily seeking to equate political and economic action beyond the fundamental claim that both can be represented as formally defined rational choice.[58]

He summarized the problem as follows:

> "In a capitalist democracy, there are essentially two methods by which social choices can be made: voting, typically used to make 'political' decisions, and the market mechanism, typically used to make 'economic' decisions. [...] The methods of voting and the market [...] are methods of amalgamating the tastes of many individuals in the making of social choices. The methods of dictatorship and convention are, or can be, rational in the sense that any individual can be rational in his choices. Can such consistency be attributed to collective modes of choice, where the wills of many people are involved?"(Arrow 1951b, pp. 1–2)

The simplest illustration is Condorcet's voting paradox. With three voters and three alternatives, each voter's preferences can be transitive, and yet majority rule yields a cycle.[59] Arrow observed: "So the method just outlined for passing from individual to collective tastes fails to satisfy

55 By contrast, Bergson's analysis relied on a more classic calculus-based maximization (Bergson 1938).

56 The better-known label is the "Impossibility Theorem." Arrow stressed, however, that his result also identified conditions under which social choice remains possible; it is therefore less pessimistic than the conventional name suggests.

57 Arrow contributed extensively to general equilibrium theory and, together with Debreu (and McKenzie), provided a foundational existence proof (Arrow and Debreu 1954; Düppe and Weintraub 2014b).

58 Arrow noted that he was not the first to compare economic and political choice. He listed authors such as Herbert Zassenhaus, Howard Bowen, and Frank Knight. His discussion also underscored that earlier comparisons often remained socio-psychological rather than formal. Arrow 1951b, pp. 5–6.

59 Given three voters, three alternatives, and the following preference orderings: A is preferred to B, and B to C for the first individual; B is preferred to C and C to A for the second individual; and C is preferred to A and A to B for the third individual (and transitivity applies ever). It is apparent that a majority prefers A to B, a majority prefers B to C, and a majority prefers C to A.

the condition of rationality, as we ordinarily understand it." (Arrow 1951b, p. 3). This is also central to the critique of welfare economics.

Arrow introduced simplifying assumptions: individual values (and preferences) are taken as given and not shaped by the analysis; and individuals are assumed to be rational. His argument did not rely on game theory, although he was aware of von Neumann and Morgenstern and recognized possible game-theoretic routes. One route concerns strategic manipulation within voting systems (for example, under plurality voting, sincere preference revelation is not guaranteed). Another concerns "games of fair division," where rules are designed so that rational play yields a fair outcome (Arrow 1951b, p. 7).

Despite his reliance on rationality assumptions, Arrow framed individual choice through "ordering relations" and pairwise comparisons among "social states." His approach therefore differs from von Neumann's Minimax framework and does not proceed via strategy sets and equilibrium existence in the same manner (Arrow 1951b, pp. 19–21). He also rejected interpersonal comparisons of utility and downplayed the welfare relevance of expected utility theory. In discussing von Neumann and Morgenstern, he wrote:

> "They consider a preference pattern not only among certain alternatives but also among alternative probability distributions. Making certain plausible assumptions as to the relations among preferences for related probability distributions, they find that there is a utility indicator (unique up to a linear transformation) which has the property that the value of the utility function for nay probability distribution of certain alternatives is the mathematical expectation of the utility. Put otherwise, there is one way (unique up to a linear transformation) of assigning utilities to probability distributions such that behavior is described by saying that the individual seek to maximize his expected utility. This theorem does not, as far as I can see, give any special ethical significance to the particular utility-scale found. [...] What it does say is that among the many different ways of assigning a utility indicator to the preferences among alternative probability distributions, there is one method (more precisely, a whole set of methods which are linear transforms of each other) which has the property of stating the laws of rational behavior in a particularly convenient way. *This is a very useful matter from the point of view developing the descriptive economic theory of behavior in the presence of random events, but it has nothing to do with welfare considerations, particularly if we are interested primarily in making a social choice among alternative policies in which no random elements enter. To say otherwise would be to assert that the distribution of the social income is to be governed by the tastes of individuals for gambling.* (Arrow 1951b, pp. 9–10, italics added)

Arrow concluded that behavior in the relevant setting could be represented with "a preference scale without any cardinal significance" (Arrow

1951b, p. 11).

Without entering into the details of Arrow's formal proof, it is important to briefly highlight his broader methodological view on mathematization. An essay Arrow published as a Cowles Commission working paper and later in a collection edited by Harold Lasswell presents this position clearly (Lasswell and Lerner 1951; Arrow 1951a). For him, the application of mathematics to natural or social phenomena rests on the notion of "model": a class of admissible structures capturing relations, from which mathematics rules out incompatible claims. Mathematics offers "superior clarity and consistency" because it forces explicitness about the components of a model, even while acknowledging the inevitable simplifications of formal representation (Arrow 1951a, 129 et ss.). It also helps address the classic tension between the individual and the collective. Following Koopmans and postwar econometrics, a full characterization of individual behavior logically implies knowledge of group behavior, while empirical analysis can consistently recover aspects of individual behavior (Arrow 1951a, p. 133). Arrow also stressed the need for statistical testing of model implications: formal modeling provides "the opportunity to tap the great resources of modern theoretical statistics as an aid in empirical validations." (Arrow 1951a, p. 133)

To conclude, let's briefly consider Downs. Like Black (though without citing him), Downs wrote as an economist seeking to explain the "rationale of government activity" using the familiar categories of producers and consumers. He described his model as "a study of political rationality from an economic point of view." (Downs 1957, p. 11) He defended the economic approach mainly by listing properties of rationality (completeness, transitivity, maximizing behavior, and intertemporal consistency) though his discussion remained largely verbal.[60] Voters are rational insofar as they maximize their political preferences, and elected officials are rational insofar as they maximize their chances of re-election.

This conception is tied to what Downs called the "self-interest axiom" (Downs 1957, p. 27). His aim was to provide a positive account of political behavior in voting and party competition. In this respect, the assumptions of rationality and self-interest function as simplifying devices for explanation. Unlike Black and Arrow, however, Downs did not engage deeply with cyclical preferences or with social choice theory more generally.

His deductive logic also supports a critique of functionalist explanations of institutions. Following Schumpeter, he stated that "social functions are usually the by-product, and private ambitions the end, of human action" (Downs 1957, p. 29).[61] Accordingly, the logic of voting follows from the self-interest axiom: citizens vote for the candidate they believe will maximize their benefits. If benefits are interpreted as utility, each citizen derives a

60 These are technical definitions. In practice, Downs's treatment of rationality and self-interest is mostly verbal.

61 Schumpeter argued that democratic politics is best understood as a "competitive struggle for power and office," which then incidentally fulfills broader social functions, in a way analogous to profit-seeking in markets (Schumpeter 1942, p. 282).

"utility income from governmental activity," and voters act to maximize that utility.

This suffices enough to show that Downs did not explicitly argue for the unity of economics and politics in the manner of Black. The contrast with Arrow is equally clear: Arrow focused on the formal equivalence of different collective choice mechanisms (markets and voting), whereas Downs concentrated on the operation of democratic institutions and the incentives within them.

These differences notwithstanding, it can safely be said that the works of Arrow, Black, and Downs helped pave the way for Riker's later agenda in political science and constituted one of the main strands from which "Positive Political Theory" emerged.