

Chapter 5

The Development of Positive Political Theory

The University of Rochester played a pivotal role in the story of how formal political theory spread into American political science, not merely as a host institution but as an intellectual community in which formal political theory could consolidate and spread. The emergence of Positive Political Theory as a recognizable subfield of American political science cannot be understood without considering Rochester's institutional strategy and its unusually strong commitment to theory-driven social science. As Sonja Amadae has emphasized, one of the distinctive features of Positive Political Theory, as opposed to Public Choice, was its close association with the intellectual leadership of a single individual. While Public Choice developed as a strongly institutionalized enterprise, first at the University of Virginia and later at Virginia Polytechnic Institute and George Mason University, Positive Political Theory was, from the outset, deeply identified with Riker's scholarly agenda and personal influence (Amadae 2003).

Thus, the development of social sciences at Rochester and Riker's intellectual trajectory are inseparable. Any attempt to assess Riker's contribution must therefore take seriously his role as an "entrepreneur of ideas," a figure who combined theoretical ambition with the ability to shape departments, curricula, and research agendas. In her historical reconstruction, Amadae also devoted considerable attention to the parallel transformation of the Department of Economics, which, under the academic and charismatic leadership of Lionel W. McKenzie, moved decisively toward a strong theoretical orientation. According to Amadae, this convergence was crucial. Riker "had a strong vantage point from which he could build a curriculum that could exert a major influence on the entire discipline of American political science because he had strong institutional support, which, not coincidentally, was similar to the advanced McKenzie's vision of economics. The University of Rochester, whose trustees firmly committed to advancing scientific methods within the social sciences, provides a compelling example of how rational choice theory was furthered as a complex of knowledge spanning several independent but interrelated fields of study" (Amadae 2003, p. 170).

There are several ways to approach this topic. One option would be to rely on the now well-established notion of "intellectual community," which emphasizes the collective and social dimensions of scientific activity. The

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history and sociology of science literature offers a rich set of taxonomies to describe such communities, often stressing the role of leadership, institutional resources, and shared research agendas (Forget and Goodwin 2011; Medema 2011). In its earliest formulation, this perspective can be traced back to the concept of the “research school” developed by J. B. Morrell, which highlights the combined importance of charismatic leadership and adequate financial support (Morrell 1972). Subsequent debates have broadened this notion considerably, stretching the idea of “charismatic leadership” from the ability to mobilize resources or attract the attention of disciplinary gatekeepers to the capacity to draw colleagues and students into a shared research program (Forget and Goodwin 2011, p. 20). In some cases, the same individual may fulfill both roles, and Riker plausibly belongs to this category.

Despite the usefulness of these conceptual tools, the discussion that follows remains deliberately historical in orientation.¹ During the Sixties and Seventies, Riker’s institutional activities at Rochester were accompanied by a broader effort to construct a network of scholars who shared his vision of formal political analysis. This period also witnessed the publication of Buchanan and Tullock’s main work and the emergence of Public Choice, as well as a series of formal innovations in political theory, including the spatial theory of voting developed by Davis and Hinich, and later contributions by Rochester-trained scholars such as Ordeshook and Shepsle.

This chapter is divided into two main parts. The first reconstructs Riker’s role in the transformation of the Department of Political Science at the University of Rochester, as well as McKenzie’s efforts in building the Department of Economics, and the diffusion of Positive Political Theory and Rational Choice during the Sixties and Seventies.

The second part of the chapter then turns to a selection of advanced theoretical developments in formal political analysis. Rather than attempting an exhaustive survey of the field, the discussion focuses on a series of debates surrounding the size principle and coalition theory. As will become clear, Riker’s analysis of political coalitions provided a set of powerful intuitions, but it was also affected by mathematical and game-theoretical limitations. By examining Robert Butterworth’s critique of the size principle, Shepsle’s response, and McKelvey’s subsequent (and largely unpublished) formal proof, the chapter highlights both the significance and the shortcomings of coalition theory as originally formulated by Riker.

¹ A general historical account of the establishment of the Department of Political Science is provided by Amadae and Mesquita 1999, partially reproduced in Amadae 2003. For the case of Economics, see D ppe and Weintraub 2014b and McKenzie 2012. Alongside secondary sources, the narrative draws on archival materials from McKenzie’s papers, housed at the Rubenstein Library at Duke University, and from Riker’s papers preserved at Rochester

5.1 Institutionalizing the "Positive Political Theory": Riker and the University of Rochester

The University of Rochester was established in 1850 in Rochester (NY). During the Second Industrial Revolution, the city emerged as a significant industrial center, where major companies such as Kodak and Xerox were founded. In addition to this industrial development, Rochester also possessed a long-standing tradition of intellectual activity. For instance, the city was home to the prominent activist and abolitionist Frederick Douglass, and it functioned as one of the forums of antislavery debate before the Civil War. Later on, Rochester was also the hometown of the social reformer and women's suffrage activist Susan B. Anthony, which provided further intellectual stimuli.

Despite this historical background, in the mid-Fifties the University of Rochester was still a relatively small private liberal arts college, located in a geographical position that was certainly not favorable. Situated in the north-western part of New York State, on the shores of Lake Ontario and close to Niagara Falls and the Canadian border, it was far removed from the intellectually vibrant environments of the Ivy League universities, the University of Chicago, or the major Californian institutions. At that time, its academic reputation rested primarily on the nationally renowned Eastman School of Music, founded in 1921 thanks to the endowment of George Eastman, the founder of Kodak.

Another philanthropist who contributed significantly to the University's academic rise was Joseph Wilson, the owner of Xerox. In the Fifties, Rochester received a substantial endowment from him, which placed it among the wealthiest research universities in the United States (Amadae and Mesquita 1999). With these resources, the University's administrators launched an ambitious program aimed at enhancing the academic status of the institution, beginning with the recruitment of senior and junior faculty and the establishment of new advanced programs of study.

One of the major consequences of these well-financed development plans was the strengthening of the social sciences. The outcome was the creation of nationally leading, theory-driven graduate programs in Economics and Political Science. In the case of Political Science in particular, Rochester eventually became the institutional stronghold of what came to be known as "Positive Political Theory." Among the first scholars hired to implement this transformation was the relatively young, yet already renowned, mathematical economist Lionel W. McKenzie.

Although McKenzie's impact on Economics was perhaps less revolutionary than Riker's on Political Science, the presence of an advanced and theory-oriented Economics Department played a crucial role in shaping the intellectual environment in which Riker could pursue his disciplinary agenda. The next pages will therefore examine McKenzie's transformation of the Economics Department, Riker's activity as chairman of the Political Science Department, and, finally, some highlights of Riker's efforts to build a broader scholarly network. Particular attention will be devoted to two aspects: his relationship with the Public Choice group, and the dissemination

of mathematical political theory.

5.1.1 The activities in the Economics Department

McKenzie (1919–2010) was undoubtedly among the most important mathematical economists of his generation. In the Fifties, his achievements were remarkable, especially given his relatively peripheral position within the community of mathematical economics. A native of Georgia and a graduate of Duke University (at a time when Duke was still a relatively prestigious regional university located in a segregated state) he later moved to Princeton, where he earned his Ph.D.² Prior to this, he had been a Rhodes Scholar at Oxford and had served as an instructor at MIT (McKenzie 1999).

McKenzie largely trained himself in mathematical economics and, in 1950, spent an entire year at the Cowles Commission at the University of Chicago, where he encountered many of the central figures of the emerging formal revolution in economics, such as Gerard Debreu, Tjalling Koopmans, and others (Weintraub 2002). Moreover, during his time in Chicago, he received systematic training in mathematics, particularly in algebra, topology, and measure theory, thereby expanding his knowledge far beyond the calculus-based preparation that still constituted, if anything, the standard requirement for students and graduates in economics (McKenzie 1999, p. 4).

After this period, McKenzie returned to Duke, where he was appointed Professor of Economics, despite not having yet received his Ph.D. His theoretical work during these years focused primarily on General Economic Equilibrium. Most notably, he produced the first mathematically rigorous, and economically meaningful proof of the existence of a General Economic Equilibrium, anticipating by several months the better-known Arrow–Debreu theorem.³

These accomplishments place McKenzie in a relatively central position within the emerging community of mathematical economists. Many colleagues recognized the originality and depth of his mathematical contributions, and he maintained close personal relationships with prominent economists, including Paul Samuelson. By contrast, while Riker was largely isolated, both within the community of political scientists, to which he formally belonged, and among game theorists, to which he aspired to belong, McKenzie enjoyed substantial recognition and esteem among his peers.⁴

Nevertheless, both his correspondence and later recollections indicate that McKenzie was only partially satisfied with his academic situation. This

² Although his dissertation had been completed earlier, personal circumstances delayed the formal awarding of the degree until 1956, after the publication of his first major theoretical contributions.

³ In addition to General Economic Equilibrium, McKenzie's contributions to economic theory also included foundational work on demand theory (McKenzie 1957) and on optimal economic growth, particularly the first of the so-called turnpike theorems. A comprehensive overview of McKenzie's theoretical contributions can be found in Mitra and Nishimura 2009 and McKenzie 2009.

⁴ For example, Samuelson wrote in a personal letter that McKenzie was among the very few mathematical economists who truly understood the economics underlying mathematical economics (cited in Duppe and Weintraub 2014b, p. 195).

dissatisfaction proved decisive in leading him to accept the offer made by the administrators of the University of Rochester in 1956. The sources of this discontent were twofold. First, on a personal level, Duke was geographically and intellectually distant from the most prestigious departments where the rapid advances in mathematical economics were taking place. McKenzie pursued a theoretical agenda that found little institutional support at Duke.⁵ More generally, the Duke faculty was not particularly receptive to the new developments in mathematical economics and, like most economics departments at the time, remained committed to traditional approaches grounded in verbal reasoning and descriptive generalizations rather than econometrics and formal mathematics. McKenzie, by contrast, strongly favored the rigorous mathematical foundations of economic theory and made major contributions in this direction. At the same time, unlike what could be described as the dominant strand of axiomatic theory in economics (most notably associated with Debreu at Berkeley, which produced some of the most abstract general equilibrium results of the Sixties and Seventies), McKenzie consistently emphasized the economic relevance of his theoretical work. For this reason, he did not regard himself simply as a mathematical economist, or still less as a mathematician working on economic problems, but rather, in his own words, as an “economic theorist.” Mathematics and rigorous analysis were essential tools for establishing logical consistency in economic theory, but they were not reducible to a mere sequence of formal theorems (Düppe and Weintraub 2014b, p. 190).

The offer from Rochester arrived in 1956, at a moment of growing dissatisfaction with the environment at Duke. As noted earlier, the University of Rochester was actively seeking promising senior scholars with original ideas to build advanced graduate programs in the social sciences, and McKenzie fit this profile particularly well. His correspondence sheds light on the recruitment process. He was offered not only a full professorship but also the chairmanship of the Department of Economics, which had recently been separated from the Business School. This proposal, together with a salary that was exceptionally high for the time, strongly appealed to him, even though he hesitated for several months, primarily because Rochester lacked the academic prestige of more established universities. Seeking advice, he consulted several colleagues, including Samuelson, who encouraged him to continue negotiations with the Rochester administration (Paul Samuelson to Lionel W. McKenzie, September 5th, 1956, McKenzie n.d., Box 6). In a letter to his friend and colleague at Duke, Frank de Vyver, McKenzie described the University of Rochester in the following terms:

"The place looks pretty good. They already have a large graduate school for the science, and the Eastman school of music and a great medical center. Also, they have one of the

⁵ Additional personal considerations also played a role, including the fact that McKenzie had married a Jewish woman in North Carolina, a state that—although more moderate than the Deep South, where he had been born, was still marked by anti-semitism, in addition, of course, to racial segregation affecting African Americans.

largest endowments of the country, and only about 1,600 undergraduates, which they expect to build up to 2,500. They really do de-emphasize athletics, and they play places like Oberlin. They have a scholarly president with a Ph.D. in history from London, who appears to be an effective leader as well. The departments of economics and business administration are merged at the moment, but they are being separated this year. I would probably go there as chairman of the Department, though I am not required to do this. The teaching load is 9 hours for about everybody with less if you are deeply engaged in research. They told me I could write my own ticket on that. In fact they painted a quite rosy picture. I would have at least two appointments available immediately." (Lionel W. McKenzie to Frank de Vyver, October 16th, 1956, McKenzie n.d., Box 6)⁶

In the same letter, McKenzie outlined both the advantages and disadvantages of a potential move. On the positive side, he emphasized the substantial increase in salary, far exceeding anything he could expect at Duke. On the negative side, he mentioned the northern climate, the administrative responsibilities associated with the chairmanship, and the challenges of building a viable graduate program from scratch at an institution lacking an established tradition in economics. In a subsequent letter written one month later, after he had accepted the offer, he stated:

"[...] The opportunity to play a central role in developing a department is attractive to me. [...] The city seems extremely nice. The university is well equipped and owns a very large endowment, alleged to be the 7th in the country. Their tuition is 850 dollars. They have about as many graduate students as Duke and only 1,500 undergraduate. There will be a couple of places to fill in the Department right away and several people are retiring soon. They have two recently acquired Princetonians, Warren Hunsberger and France (in labor). France seems very promising. He has done some writing with the industrial section in Princeton. Anyway it will be an adventure." (Lionel W. McKenzie to Frank de Vyver, November 23, 1956, McKenzie n.d., Box 6)

As noted above, McKenzie was recruited with the explicit mandate of developing a first-rate graduate program in economics. One of the initial steps in this process was the formal separation of the Department of Economics from the Business School, a decision that had been made prior to his arrival but only implemented after his appointment but was fully consistent with his views on economics education. Indeed, At Duke, he had advocated precisely such a separation between business and economics programs (Düppe and Weintraub 2014b). Even more important, however,

⁶ McKenzie is referring to the historian Cornelis de Kiewet (1902-1986), previously president of Cornell University

was the hiring strategy he pursued. At the time of his arrival, the department consisted of three full professors, one associate professor, one assistant professor, and one lecturer, none of whom were closely engaged with the most advanced developments in economic theory.⁷ Their research fields reflected the standard undergraduate curriculum in U.S. economics at the time, including money and banking, public finance, international economics, area studies, labor economics, government, and economic history (Albert W. Noyes to Lionel W. McKenzie, November 20th, 1956, McKenzie n.d., Box 6). Immediately after accepting the position, McKenzie began searching for young scholars to recruit. Even before formally accepting the offer, he had outlined his hiring standards in a letter to Noyes, the Dean of the College of Arts and Sciences. In that letter, McKenzie wrote that he “[...] would like to have two appointments available in the beginning, preferably one at the associate level and, if the right man is available, one at the full professor level.” (Lionel W. McKenzie to Albert W. Noyes, October 29th, 1956, McKenzie n.d., Box 6).

Between late 1956 and early 1957, McKenzie wrote to numerous economists with whom he was well acquainted, including Samuelson, Robert Dorfman, Arrow, William Baumol, Milton Friedman, Jacob Viner, Franco Modigliani, and Albert Hirschman. He sought advice on recruiting graduate students from their departments and requested assessments of the existing Rochester faculty. Among these efforts was an unsuccessful attempt to recruit Gary Becker, who ultimately chose to remain at the University of Chicago (Lionel W. McKenzie to William Baumol, November 28th, 1956; Lionel W. McKenzie to Milton Friedman, November 28th, 1956, McKenzie n.d., Box 6).

The first new faculty appointment was Ronald Jones, who had earned his Ph.D. at MIT under Robert Solow and would later become one of the leading figures in international trade theory (Düppe and Weintraub 2014b). He was soon followed by Richard Rosett, Edward Zabel, Nanda Choudry, Michio Hatanaka, S.C. Tsiang, Rudolph Penner, Alexander Eckstein, Sherwin Rosen, and Robert Fogel. Fogel later moved to Chicago and was awarded the Nobel Prize in Economics in 1994 for his contributions to quantitative economic history. One year later, the graduate program was launched, with a strong emphasis on recruiting students from abroad, particularly from Japan, Europe, and other parts of Asia. Among the earliest cohort of students, Akira Takayama would later emerge as a prominent mathematical economist.

Further insight into the department’s scientific activity in the early 1960s is provided by reports prepared for the Committee on Social Science (CSS) at the University of Rochester. These documents offer a broader overview of the state of social science research at Rochester and of the plans devised to promote its development. The Committee was established in the fall of 1961 and included representatives from all social science departments in the College of Arts and Sciences: Anthropology, Political Science, Psychology,

⁷ These faculty members were William E. Dunkman, Donald W. Gilbert, Warren Hunsberger, Robert R. France, Jack Taylor, and Alice Wynd.

Sociology, History, and Economics. Drafts from the first meetings, held between October and November, reveal that the central concerns included interdisciplinarity and the alignment of Rochester's social science programs with national research trends. The reports also describe the activities, objectives, and scope of research pursued by each department.

The CSS reports outline how research in the Economics Department was structured under McKenzie's chairmanship. Scholarly activity was divided into empirical and theoretical work. Empirical research included, for example, Fogel's investigations into American economic history, with particular attention to the role of railroads. Methodologically, Fogel framed his work "within the categories of growth economics," employing advanced mathematical and statistical techniques to collect and analyze data. The same report emphasized the expansion of funding and infrastructure necessary for such empirical research, especially computing facilities and library resources ("Empirical Research currently under way in the Department of Economics," McKenzie n.d., Box 32). Notably, even though economic history constituted only one component of empirical research, particular emphasis was placed on the "continued development of relations between the economics and history departments" ("Empirical Research currently under way in the Department of Economics" 1960). A similar call for interdisciplinarity was made with respect to political science, especially in areas related to taxation and fiscal policy. Research in theoretical economics, McKenzie's primary field, covered topics ranging from the mathematical analysis of competitive markets, with special emphasis on the key issues surrounding general equilibrium (existence, uniqueness, and stability), to international trade and macroeconomics. Among the broader objectives identified was also the development of mathematical and statistical methods in the social sciences, with particular attention to choice behavior, psychology, and social psychology ("Research in Theoretical Economics" 1960, McKenzie n.d., Box 32).

Although these reports are largely descriptive and remain at a general level, they are invaluable for understanding the kind of commitment to social science research and interdisciplinary collaboration that characterized Rochester in the early Sixties. Even without extensive detail, they convey the image of a highly ambitious and intellectually stimulating academic environment, an impression corroborated by interviews, personal recollections, and oral histories of scholars who spent part of their careers there. As D uppe and Weintraub observed: "By the early 1960s, McKenzie had built up the Rochester research community with its own identity, distinct from that of Berkeley, and proud to be labeled theoretical as opposed to mathematical. [...] Rochester welcomed mathematics and science 'majors' into its graduate program as over the 1960s it became known as an exciting place for them to study economics and political science." (D uppe and Weintraub 2014b, p. 193)

5.1.2 Setting the "Positive Political Theory"

In the late Fifties, the Department of Economics was undergoing significant transformations, and the Department of Political Science was on the eve of similar changes. It is important to note that the Committee on Social Science mentioned above was established several months before Riker officially assumed his position at the University of Rochester. Even so, the discussions concerning Political Science were clearly shaped by his impending arrival. Indeed, the meeting drafts make it evident that Riker was hired primarily for his ideas about how the discipline ought to develop. Moreover, although he was not yet formally on the faculty, many references, albeit very general ones, were made to the activities he was expected to undertake in the Department.

So then, in the first draft describing the situation in the Political Science department, it is stated: "Political Science, though not distinguished for its research, provides good teaching at the undergraduate level. A number of recent additions to the Department have increased the research potential. A new chairman, Professor Riker of Lawrence College, assumes his responsibilities in September 1962. There are plans to increase the budget and to build gradually to a ten-man department." (University of Rochester 1961, October 2nd, 1961) Then, even though the Department was represented on the Committee by Richard Fenno, a scholar committed to traditional analysis (though not hostile to a more formal approach), it nonetheless appears that some of Riker's ideas found their way into the discussions. In particular, emphasis was placed on the new perspectives Riker was expected to introduce, namely the "growth toward a doctoral program," beginning with the "acquisition of men with broad theoretic interest and with skill in the application of scientific analysis to the subject matter of politics." (Appendix to the minutes of "Committee on the Social Sciences," October 2nd, 1961, McKenzie n.d., Box 32). Again, at the third meeting, when Fogel and McKenzie discussed empirical research and applied mathematics, Fenno remarked on Riker's active interest in mathematical methods for the study of political behavior. (University of Rochester n.d., November 6, 1961, McKenzie n.d., Box 32).

As noted above, each department was also asked to prepare a report for the Committee. Unfortunately, the report on Political Science is unsigned, so it is impossible to determine whether it was written by Riker or instead by Fenno. Nonetheless, even a cursory reading suggests that Riker's theoretical agenda is visible once again. With regard to teaching, the Department followed a traditional and fairly standard curriculum: American Politics, Comparative Political Systems, International Politics, and Political Theory. Yet the report also anticipates that "beginning next year [...] we will be giving a different, more theoretically and broadly oriented introductory course". ("Report on Political Science for Committee on Social Science" 1960 Report on Political Science for Committee on Social Science, in McKenzie n.d., Box 32) This reorientation of teaching was presented both as a prerequisite for creating a graduate program and as a means of reshaping departmental activity more generally. This was particularly urgent because,

in the closing years of the Fifties, the Department “was really going downhill in terms of its undergraduate activities,” since, as Riker recalled to Kenneth Shepsle, in 1957 “no people decided to major in political science. So that there were no seniors in political science in 1959, which was the nadir of the department.” (Riker and Shepsle 1979, p. 62)

Among the research projects the Department intended to pursue were the analysis of decision-making in small groups, the study of federalism, the analysis of congressional committees, and also more explicitly theoretical topics such as “methods and hypotheses which may be used to develop a general theory of politics” or “the relationship between leisure behavior and political behavior.” The report also acknowledged the intrinsically interdisciplinary character of these projects, for instance, the connections among decision-making theory, mathematics, and economics (“Report on Political Science for Committee on Social Science” 1960).

As for hiring policies, the stated goal was explicitly to recruit “men of broad theoretic interests and with skill in the application of the important analytical techniques of social science to the subject matter of politics.” (*ibidem*) This necessarily implied an interdisciplinary approach, with a broader research horizon that could be sustained only through strong collaboration with other fields. However, as the report stated, “[...] it is difficult to be precise about the nature of all our needs. Certainly, all existing strength in the social sciences plus Mathematics and Philosophy will be a source of strength to Political Science.” At the same time, the emphasis was not exclusively on formal analysis and mathematical techniques. For example, social psychology was also considered part of this broader renovation effort.

The report concluded with a general statement about the future of the discipline: “the future of Political Science as a social science depends upon its ability to link systematic theory about human behavior and accurate descriptions of distinctly political activity. Moves which promote this ‘marriage’ - within or without the Department - will enable Political Science at Rochester to operate at the frontiers of the discipline. Our goal is to do this and to do so with distinction.” (*ibidem*)

Riker arrived at Rochester in 1962. He was hired by Noyes in 1961 under conditions very similar to those offered to McKenzie, and he began his work in the fall of 1962. Like McKenzie before him, Riker was seeking new professional opportunities and was partially dissatisfied with his previous academic environment. Moreover, McKenzie almost certainly played a role in persuading Riker to join Rochester. As McKenzie wrote: “I recall my discussion with Bill Riker when he came to the University to consider whether to accept a position there [...] Of course I welcomed him heartily to Rochester and described my Department’s goals to him. The presence of an economics department whose success was already well known played an important role in making these appointments possible.” (McKenzie 2012, p. 232)

Riker formally assumed his appointment as full professor of Political Science and chairman of the Department in the fall of 1962. As soon as he arrived, he began a program of radical renovation, closely following the lines discussed above. At Rochester, he found precisely the environment

he needed to advance his agenda, and he was supported even by scholars, such as Richard Fenno, who remained committed to traditional approaches. Riker recalled that, beyond personal reasons, what attracted him were the efforts made by the Rochester administration to move “from a small liberal arts college to a university and systematically developing departments” and to build an advanced graduate education program (Riker and Shepsle 1979, p. 62).

No one resisted his plan to improve the Department’s standing by advancing his own vision of political science, one that, plainly, required teaching courses in decision theory and game theory, as well as methodology. Shepsle, who entered the newly established graduate program in 1967, recalled that “when [Riker] came to Rochester, he came as the chair of the Department. Quite innocently, he sent around a memo to his colleagues proposing a Ph.D. program where he emphasized game theory and decision theory, and econometrics. He expected to have a broad discussion with these colleagues about it. His colleagues immediately said yes. And so he began to build the faculty that could teach that kind of political science.” (Shepsle 2021) What Riker aimed to accomplish can be succinctly captured in the brief memorandum on Political Science curriculum that he drafted as a Fellow at Stanford’s “Center for Advanced Studies in the Behavioral Sciences” (1961).⁸ The same vision that can be found throughout his work in the late Fifties, culminating in *The Theory of Political Coalitions*.

To understand Riker’s perspective, and how it reshaped the teaching of political science at Rochester, some attention must be paid to the Department’s curriculum and broader institutional condition. Rather than relying solely on the Committee on Social Science drafts, a more complete picture can be drawn from the University of Rochester Undergraduate Bulletins. These bulletins provide, alongside a general description of the University’s history and activities, a detailed list of departments, faculty members, and courses. For the academic year 1961–62, the Political Science curriculum reflected a traditional structure, divided into “International Relations,” “American Politics and Institutions,” and “Political Theory.” The latter also included an introductory course in “Scope and Methodology in Political Science.” (“University of Rochester Undergraduate Bulletin 1961-2” 1961, pp. 147–9) Riker arrived the following year, and among his first hires were Arthur Goldberg and, most importantly, Gerald H. Kramer as an instructor, who joined the original six members of the faculty (in addition to Riker himself).⁹ Kramer, an MIT Ph.D., had a strong mathematical background, and his role would prove essential in the following years for advancing both training and research in mathematical political science within the Department. After Riker’s arrival, the number of undergraduate courses in scope and methodology doubled, and by the academic year 1963–64 a course

⁸ In that memo, Riker explicitly stressed the attention on positive methods of verifying hypotheses and positive theories of politics, whereas ‘positive’ assumed the meaning of non-normative theories (Amadae and Mesquita 1999). See also the discussion earlier

⁹ These were Richard Fenno, Theodor Bluhm, Glen Wiltsey, William Diez, Dale Neuman and Peter Regenstreif (Riker and Shepsle 1979, p. 63)

titled “Positive Political Theory” had been introduced within the “Political Theory” section (“University of Rochester Undergraduate Bulletin 1963-4” 1963). Amadae and Bueno de Mesquita described the transformation of Rochester political science as follows: “whereas other programs emphasized the literature, Riker focused on developing tools for rigorous research into the theoretical properties and empirical law of politics.” (Amadae and Mesquita 1999, pp. 279–80). Within this approach, the notions of rational choice in political decision-making clearly occupied a central place.

Still, as in the case of the Department of Economics, a crucial prerequisite for establishing a successful graduate program was the recruitment of both faculty and students. As Riker later recalled:

"I devoted the first year to finding teachers to expand [...] and also outlining and planning a Ph.D. program. [...] I was extremely fortunate to hire two people who were entirely sympathetic with the kinds of things I wanted to accomplish in that Department. One was Jerry Kramer and the other was Art Goldberg. [...] Kramer was the first person that I could find who was both a political scientist and was interested in teaching about statistics. [...] And it was pure luck to be able to find him. And I think that his experience here was very good for him because he came here thinking of his role in the world as being a statistician. And, in the five years or so he spent at Rochester, I think he changed his vision of himself to being a political scientist and being interested in political theory. And I always felt that that was one good thing that this Department did for the world was to make Jerry into a political theorist." (Riker and Shepsle 1979, p. 65)

Kramer, according to Shepsle, was “probably the leading mathematical political scientist at the time” (Shepsle 2021) and played a pivotal role in supervising the dissertations of analytically oriented graduate students such as Peter Ordeshook, Shepsle himself, and especially Richard McKelvey. After his period at Rochester, Kramer joined the faculty at Yale and spent substantial time at the Cowles Foundation there.

Beginning in 1967–68, Richard Niemi joined the faculty. He earned his Ph.D. at Michigan State but had earlier been an undergraduate at Lawrence, where in 1961 he had likely attended one of the earliest courses taught by Riker on “Positive Political Theory,” during the same period in which Riker was also a Fellow at Stanford (Niemi 2021). Although he lacked formal training in mathematical modeling, unlike Kramer or several of the graduate students who began arriving once the graduate program was underway, Niemi had strong statistical training and fit well within the theoretical agenda Riker was pursuing. Accordingly, by the academic year 1967–68, the number of undergraduate courses addressing topics such as game theory, decision theory, and analytical methodology had risen to five (“University of Rochester Undergraduate Bulletin 1967-8” 1967).

Nevertheless, the most important development in the second half of the Sixties was the definitive establishment of the graduate program, which

was precisely the task for which Riker had originally been recruited. The first student enrolled in 1962, but the program officially began in 1963. Ordeshook, who entered with the class of 1964, recalled that although he chose Rochester over Columbia largely by chance, he was deeply struck by Riker's treatment of formal political theory (Ordeshook 2021). The curriculum covered virtually all major topics available at the time, ranging from Arrow's Impossibility Theorem to spatial models of voting, and including Buchanan and Tullock's *The Calculus of Consent* as well as Olson's newly published *The Logic of Collective Action* (1965). It also addressed Riker's work on political coalitions and game theory, using Luce and Raiffa's textbook (Luce and Raiffa 1957). At the same time, as Ordeshook wrote to the author, "*Riker was interested in such things only to the extent that they said something about real politics. Riker's philosophy, then, was quite simple: The emerging field of formal political theory should not spin off into arcane mathematics or a plethora of axioms but instead address classical political questions.*" (Ordeshook 2021. Italics in the text)

Shepsle, who arrived in Rochester in 1967, similarly recalled that Riker was deeply involved in graduate training. He taught two seminars required of every graduate student: an introductory seminar on game theory and another called "Positive Political Theory." (Shepsle 2021) A more advanced game theory course was likely taught by Gerald Kramer, whose mathematical abilities exceeded those of virtually anyone on the Rochester faculty at the time. However, students, according to Ordeshook and Niemi, were also required to take traditional courses, including those taught by Fenno and Bluhm. In any case, by the late Sixties, the Rochester graduate program was widely recognized for its strong emphasis on mathematical political science and formal modeling. One consequence was that the undergraduate background of applicants differed substantially from that of students in other political science programs. As Shepsle put it: "When I came to Rochester, it was still a collection of oddballs in a way who ended up in graduate school at Rochester, people who had done work in physics, or in engineering, or in mathematics, who gravitated normally as undergraduates into political science and then on to Rochester." (Riker and Shepsle 1979, p. 70) Shepsle had majored in mathematics, while, as noted, Ordeshook was trained as an engineer. The 1968 cohort included Richard D. McKelvey, who had a similar background (Palfrey 2005), and this pattern continued throughout the Seventies.

Mathematical reasoning occupied a central place in "Positive Political Theory." Indeed, as Riker conceived it, this was not merely an "economic approach," in the sense of the many parallels historically drawn between economic and political action.¹⁰ Rather, at its core lay the importance of formally modeling political phenomena, in which concepts such as preferences, choice, and equilibrium acquired meaning precisely through the internal consistency of the model. For this reason, mathematical training played a pivotal role in the education of political scientists at Rochester. It should be noted, however, that Riker was not really a mathematician.

¹⁰ For a brief discussion of this intellectual history see: Barry 1970

As mentioned earlier, he was largely self-trained, and his technical skills were well below those required to pursue the ambitious theoretical agenda he had set for himself. As Bruce Bueno de Mesquita remarked, “[Riker] had not many mathematical skills, but a great mathematical intuition. He had a good understanding of how mathematics can be used by people with much more training than he had. Of “Political Science,” he put emphasis on the word “Science.” But he certainly was not a mathematician.” (Bueno de Mesquita 2021) With respect to mathematical training for graduate students, several options were initially available. On the one hand, scholars such as Kramer taught them; on the other, students were encouraged to take econometrics and statistics courses in the Department of Economics or in Mathematics (Ordeshook 2021; Shepsle 2021). For example, after he joined the Economics Department, James W. Friedman, one of the most important contributors to game theory in the Seventies, helped political science students acquire more solid game-theoretical training, as Ordeshook recalled (Ordeshook 2021). Moreover, because the graduate program attracted students with strong mathematical backgrounds, extensive remedial training was often unnecessary, at least in the earliest years. As Bueno de Mesquita recalled: “In the early years’ many students came in with good mathematics background, Dick McKelvey came with a lot of math, but he was clear that was not going to be perpetually true, so Bill established a course often taught in the summer, a math course for political science. The essentials of calculus, matrix algebra, and linear algebra. The graduate students who were interested in modeling, of course almost everybody had to come with math, or to take a math course.” (Bueno de Mesquita 2021) It is remarkable, however, that no formal training in economics seems to have been required, let alone in mathematical economics. What mattered was mastering the basic techniques, namely mathematics and statistics, rather than falling prey to any supposed “imperialism” of economic science.

5.1.3 The Spreading of Formal Political Theory in the Sixties and early Seventies

In 1962, the same year of Riker’s appointment, the University of Rochester hired the renowned statistician and economist Wilson Allen Wallis. Wallis had previously been the Dean of the University of Chicago Graduate Business School and came to Rochester to assume the chairmanship of the University, eventually also becoming chancellor between 1970 and 1976. Under Wallis’ mandate, the University completed its transition from a small regional university to a leading U.S. research institution. Moreover, Wallis was not merely a gifted administrator. Rather, he was embedded in an extended network of scholars and public intellectuals. At the University of Chicago, he had established strong ties with figures such as the future Nobelists Milton Friedman and George Stigler. He was also a member of the Mont Pèlerin Society, the association of liberal and free-market scholars established after the Second World War by F.A. von Hayek.

Teaching at Rochester was pivotal for establishing Positive Political Theory during the Sixties. In the same period, Riker was also working to build a network of scholars who shared a similar theoretical agenda. In

Virginia, Buchanan and Tullock adopted Price Theory and Cost Analysis to address issues ranging from public finance to voting theory, and their activities paved the way for the establishment of Public Choice. Throughout the decade, a series of meetings, academic conferences, and training programs, sponsored both by academic institutions and by organizations such as the National Science Foundation and the Social Science Research Council, were organized. Indeed, in the Sixties, after the publication of the “founding text” of Public Choice, *The Calculus of Consent* (Buchanan and Tullock 1962), Buchanan and Tullock began a series of conferences devoted to “non-market decision making.” Similarly, at the beginning of the decade, the Department of Government at the Southern Methodist University in Dallas hosted a series of conferences on “Mathematical Application in Political Science,” beginning in the summer of 1964 and lasting until the early Seventies. There, the emphasis was placed on theoretical contributions, such as voting models, as well as on statistical analyses. Eventually, these conferences moved to the Virginia Polytechnic Institute in 1966, since their organizer, Joseph L. Bernd, had relocated there. Still, Virginia Polytechnic was the hometown of Public Choice, thanks to the presence of Buchanan (Levy and Peart 2020). In addition, third and fourth conferences on “non-market decision-making” were held in New York and Chicago, organized by Riker in 1966 and 1967, under the patronage of the Social Science Research Council.

In the summer of 1963, when he was still an Associate Professor of Political Science at Southern Methodist University, Bernd invited Riker to participate in the planned conference on Mathematical Applications in Political Science, to be held the following year. In his invitation letter, Bernd presented the conference as an opportunity for post-doctoral researchers to expand their training in formal techniques in political science and to consult on research projects. Accordingly, Riker was asked to present a paper on Game Theory and legislative policy (Bernd to Riker, July 18th, 1963, Riker n.d., Box 11, Folder 2). At that conference, Riker met Downs, who was among the lecturers. However, only the lectures by Riker and Downs addressed theoretical aspects. Riker’s lecture focused on cyclical majorities, revisiting themes from his 1961 paper. Downs, by contrast, offered a general appraisal of rationality in politics and political modeling (“Program of the Conference on Mathematical Applications in Political Science,” Riker n.d., Box 11, Folder 2). Riker’s lecture was later published (in revised form) in the first conference proceedings (although Downs’s did not). In the published version, Riker reassessed Arrow’s paradox, focusing on two episodes in which cyclical majorities had occurred: one was the case of the “Powell amendment” in 1956; the second, already discussed in earlier work (Riker 1961), was the failure to pass the 17th amendment to the U.S. constitution in 1911 (direct elections of the U.S. Senate).¹¹

¹¹ The Powell Amendment case became an example to which Riker often returned. In a nutshell: the issue at stake was a bill for school construction proposed to the House of Representatives in 1956. The bill failed when it was proposed to limit the aid only to desegregated schools (the “Powell amendment”). According to Riker’s interpretation, this was an example of a cyclical majority. A majority of representatives were in favor of the bill (both Republicans and Democrats). Those

The other lectures at the conference dealt with statistical topics used to analyze political behavior, both in domestic politics and in international relations, and were delivered by renowned quantitative scholars such as Harold Guetzow, Donald Stokes, and Sidney Ulmer. Overall, thirty-five scholars attended the first conference between July 19th and 29th, 1964 (the average age was 39: Bernd to Riker, March 16th, 1964, Riker n.d., Box 11, Folder 2). With few exceptions, the applicants lacked advanced mathematical and statistical skills. This was also because, as the organizer noted, priority was given to scholars without advanced training in these techniques, for the obvious reason that this group was far more numerous within the political science profession.

Riker was invited to participate in the second conference as well. In the remainder of the Sixties, these scholarly meetings and the ensuing proceedings catalyzed the exchange of ideas and the development of new, increasingly formal political theories. Moreover, they fostered the “meeting of minds” that made the diffusion of formal political theory possible (Bernd, Claunch, and Herndon 1964-73; Medema 2009). Furthermore, from the second conference onward (1965), statistical analysis also opened the way to a growing number of theoretical and even methodological contributions. Here, Riker’s influence is readily visible, as his correspondence with Bernd makes clear. Indeed, Riker conveyed to him his concerns about the state of political science, which he regarded as “far behind Economics in quantification,” with the discipline “hav[ing] had no Alfred Marshall.” (Bernd to Riker, July 31st, 1964, Riker n.d., Box 11, Folder 2) Ultimately, the second conference (July 18th–August 7th, 1965) included a decision-theoretic analysis applied to political campaigning (specifically, investigating whether door-to-door canvassing maximized each candidate’s expected plurality; Kramer 1966); a reassessment of the size principle by Riker; and, finally, a landmark paper by Otto Davis and Melvin Hinich (Carnegie Mellon), in which they addressed policy formation within an advanced theoretical framework (Davis and Hinich 1966).

Given the importance of the latter contribution, it is worth spending a few words on it. Davis and Hinich’s work offered the first mathematical exposition of the spatial preferences model. As noted, the idea of spatial preferences in politics dates back to the work of Black, Downs, and, earlier still, Hotelling. Yet their contributions had not explored the problem in depth from a mathematical standpoint. Black restricted his discussion to single-peaked preference functions, whereas Downs largely overlooked the issue, focusing instead on the probability distribution of voters’ preferences over a single-issue space.

Both Davis and Hinich had not been trained as political scientists. Davis earned a Ph.D. in economics at the University of Virginia, where he studied under James Buchanan. Hinich obtained his Ph.D. in statistics from Carnegie Mellon (and he also held a BS and MS in mathematics) (Hinich

who were against it (mostly Republicans) favored the introduction of the amendment in order to split the opposite group, inducing the southern Democrats to oppose the bill. As a result, the amended bill failed to pass (Riker 1969; Riker 1982).

and Munger 2006; Jones et al. 2011). Before the two met at Carnegie-Mellon, Hinich had spent many years working as an applied mathematician for research institutions such as Bell. At Carnegie, Davis was among the promoters of the School of Public Policy and Management, established in 1968. Hinich also worked at Virginia Polytechnic and, later, at the University of Texas in Austin.

The central achievement of the 1966 paper was the formalization of the spatial theory of voting and elections. In doing so, the authors specified Downs' theory more precisely, showing why it is a dominant strategy for each candidate to adopt the median voter's position in the case of a single policy issue. They also extended this result to the case of n -issues, showing that when the density of preferred points is normal, the dominant strategy consists of choosing a policy vector that coincides with the vector of the means of voters' preferred positions. Another result of the paper was the finding that, given two parties, if the variance of the density of preferred points for the first party is much smaller than for the second, the first party may win a plurality even though it is the smaller party (Davis and Hinich 1966, pp. 189–95).

Beyond these formal results, the importance of their analysis lay in the formalization of Rational Political Choice and in a basic generalization of the idea of "Political Space." The problem to be investigated, in the authors' words, was the following:

"Given the precisely defined [...] and unchangeable preferences of the voters in the population, candidates for public office compete for votes by announcing before the election their exact position on each of the relevant issues. Each voter compares the position taken by the various candidates and casts his vote for that particular candidate whose position is "nearest" [...] his own most preferred position. It is assumed that once elected, a (former) candidate will adopt those policies which he announced during the campaign. Thus the questions to be answered are whether, and under what conditions, dominant strategies exist for the candidates." (Davis and Hinich 1966, p. 176)

Assuming, first, that policies can be measured by certain indices and, second, that these indices can be normalized across voters, Davis and Hinich represent each voter i 's preferred position as a vector, $\vec{x}_i = [x_1, x_2, \dots, x_n]'$. Similarly, each candidate j 's preferred policy can be represented by a vector. Each voter can then construct a loss function measuring the distance between his preferred point and a candidate's preferred position:

$$L_i(\theta) = (\vec{x}_i - \vec{\theta})' A (\vec{x}_i - \vec{\theta})$$

This is a quadratic utility function, where A is a symmetric positive definite matrix. The idea of Rational Political Choice can therefore be stated precisely by observing that each voter's loss function "has an obvious relationship to the economist's notion of a utility function," because it incorporates the idea of declining marginal utility, one of the foundational

notions of utility analysis. Each voter then chooses the candidate who minimizes her loss function (Davis and Hinich 1966, p. 178). Though fairly straightforward, these assumptions provide an important point of departure for formally characterizing individual political choices. With two candidates, 1 and 2, each voter i casts his vote for candidate 1 if $(\vec{x}_i - \vec{\theta}_1)'A(\vec{x}_i - \vec{\theta}_1) < (\vec{x}_i - \vec{\theta}_2)'A(\vec{x}_i - \vec{\theta}_2)$. The authors then showed that when $n = 1$, the dominant strategy for each candidate is to adopt the median voter preference: $\theta_j = \theta^*$.¹²

The idea of an individual loss function was borrowed from statistics. However, it can easily be generalized within a mathematical model: for any n alternatives, a choice function exists that maps preference into a Euclidean space \mathbb{R}^n . Accordingly, the utility function corresponds to the Euclidean distance between a voter's preferred alternative and another one. This is the fundamental mathematical assumption on which the formal developments of Spatial Voting Theory from the Seventies onward are based, including two of the most important results in highly formal political science: the extension of the median voter theorem to multidimensional issues (Davis, DeGroot, and Hinich 1972); and the proof of the persistence of intransitivities in multidimensional voting models (McKelvey 1975; see also Austen-Smith 2006)

That a foundational contribution such as Davis and Hinich's 1966 paper was presented at one of these conferences on Mathematical Political Science is evidence that the efforts of Riker and his affiliates were beginning to bear fruit. Returning to the conferences themselves, it is also important to stress that, beyond strictly theoretical contributions, the foundations of the models were themselves a topic of discussion. For instance, at the fourth conference, organized at the University of Virginia, where Bernd had moved, Otto Davis explored methodological issues that complemented those addressed by Riker in his early work (Davis 1968).

After Bernd moved to Virginia Tech, it became the host of the subsequent six conferences on mathematical political science, which continued until 1973. Eventually, both Davis and Hinich joined the "Public Choice Society" (first established as the "Committee on non-market decision making" and later renamed the "Public Choice Society" in 1968) and served as its president (Davis between 1970 and 1972. Hinich between 1992 and 1994). Before them, Riker himself held the same position: his appointment in 1966, immediately after the mandates of Buchanan (1964) and Tullock (1965), shows how close the Rochester group was to the activities of the Virginia group.

¹² This is nothing but the "median voter" theorem, now demonstrated in a precise way. Given $n = 1$, each voter's loss function is simply the distance between his preferred position and each candidate's preferred position: $|x_i - \theta_1| > |x_i - \theta_2|$ (or the contrary). Consider θ^* , that is the median preference, which satisfies the following conditions: $P(x \leq \theta) \leq \frac{1}{2}$ and $P(x \geq \theta) \geq \frac{1}{2}$. Suppose the first candidate chooses the median position, and the second chooses $\theta_2 > \theta^*$. Then, since the former obtains for sure the preference of at least $\frac{1}{2}$ of the total of the voters, each strategy is dominant. Clearly, to try to win the election, the second candidate also needs to select a median platform (Davis and Hinich 1966, pp. 181–2).

While there is no doubt that the impact of Public Choice on postwar social sciences was far more significant than that of Positive Political Theory (especially after James M. Buchanan was awarded the Nobel Prize in Economics in 1986), their development partially overlapped, and the two subfields ran in parallel and grew together.¹³

Four years after the first meetings of the Committee on Non-market Decision-Making,” together with the name change to “Public Choice Society,” the eponym journal was created (1968). The journal evolved from the “Papers on Non-Market Decision-making,” a review that lasted between 1966 and 1967 and originated from a series of conferences organized in 1963 and 1964 by Buchanan, with Riker as a guest. Riker positively appraised the work of Buchanan and Tullock and favorably reviewed *The Calculus of Consent* in 1962 (Riker 1962a). Moreover, in their work, Buchanan and Tullock also analyzed simple majority voting through game theory in characteristic-function form, albeit in an elementary model. In doing so, it is apparent that they benefited from Riker’s comments (Buchanan and Tullock 1962, p. 150).

The principal difference between the two approaches lies in Buchanan and Tullock’s normative stance (even if their analysis also included a positive dimension) and in its connection to the analysis of public finance from a theoretical perspective. Moreover, Buchanan and Tullock ultimately favored Chicago-style price theory over game theory and mathematical modeling (even if in their 1962 work they briefly explored some game-theoretical notions regarding n -person games).

In 1966, Mancur Olson, who the year before had published his pivotal analysis of collective choices (Olson 1965), submitted to the Social Science Research Committee a request to create an interdisciplinary committee to organize a series of conferences on non-market decision-making (Medema 2000). This proposal paralleled a similar initiative advanced by Riker in August 1966 (Bryce Wood to Riker, September 20th, 1966, Riker n.d., Box 29). Unfortunately, both requests were rejected, but the Research Committee opted to sponsor a new conference and, in particular, a summer training program in mathematical political science.

Riker served as chairman of the fourth Conference on Non-Market Decision-making, held in Chicago on 8–9 December 1967. During the two-day conference, participants discussed an econometric model of fluctuations in U.S. voting behavior developed by Gerald Kramer, as well as a debate concerning the frequency of voting paradoxes. Finally, sociologist James Coleman presented a paper on the marginal utility of voting (Coleman 1968). The list of invited participants gives a sense of the network Riker had assembled by the late Sixties. Among others, it included familiar names such as Davis, Downs, Olson, Kramer, and Ordeshook; cutting-edge political and social scientists such as James Coleman, Vincent Ostrom, Hayward Alker, and Philip Converse; and economists and philosophers such

¹³ For a history of the genesis and development of Public Choice, see Medema 2009; a highly critical (and criticized) perspective is offered by: MacLean 2017; see also Boettke 2019; Levy 2017.

as Oliver Williamson, John Rawls, and the game theorist John Harsanyi (McKenzie n.d., Box 29).¹⁴

A summer course in Mathematical Political Analysis was held in 1968, jointly sponsored by the Social Science Research Council and the Inter-University Consortium for Political Research (ICPR), at the University of Michigan, Ann Arbor. The course was divided into two parts. The first, directed by Riker, emphasized the main issues of the emerging “Positive Political Theory,” such as Spatial Voting Theory and Game Theory. The second, directed by Kramer, examined advanced econometric techniques for political model building (“Report to MSSB on Summer Course in Mathematical Political Analysis,” Riker 1968), McKenzie n.d., Box 29). This course is interesting for two main reasons: first, as Riker wrote in the report submitted to the “Mathematical Social Sciences Board” within the Social Science Research Committee¹⁵: “almost all the participants asserted that the course covered materials not available in the curriculum of their home schools [...]”; second, all the teaching staff (Riker, Kramer, and Riker’s graduate student William Zavoina) were part of the Rochester political science community, so that this course can be interpreted as “Positive Political Theory” setting foot in a place, the University of Michigan, that was the stronghold of the quantitative (but not theoretical) approach to political science associated with the behavioral revolution.

Taken together, these initiatives portray a vibrant intellectual community centered on Rochester, but with additional layers in Virginia and, especially, Carnegie Mellon, where several Rochester Ph.D.s were recruited (notably Ordeshook and McKelvey). According to Ordeshook, Carnegie Mellon was as crucial to the development of formal political science in Riker’s mold during the 1970s as Rochester itself. It was also the institution where Herbert Simon spent most of his career and where he was working when he received the Nobel Prize in Economics in 1978. However, Simon had virtually no influence on political science there. A decisive role was instead played, beyond Davis and Hinich, by the economist Allen Meltzer, Dean of the Business School and himself interested in the causes of government growth, and by William Cooper, who succeeded Davis as Dean of the School of Urban and Public Affairs and recruited McKelvey once he completed his Ph.D. with Riker in 1974 (Palfrey 2005; Ordeshook 2021).

At Carnegie, the group composed of Ordeshook, McKelvey, Davis, Hinich, and Howard Rosenthal (a political scientist and another MIT Ph.D.), James Lange (a sociologist), and Thomas Schwartz (a social choice theorist) produced some of the most important contributions of the second generation of work in “Positive Political Theory,” ultimately completing its transformation from the pioneering, but not adequately formal, early contributions of Riker into a fully fledged axiomatic and mathematical research field.¹⁶ In 1978, McKelvey moved to Caltech, establishing the last,

¹⁴ Both Harsanyi and Williamson future Nobelists in Economics, respectively in 1994 and 2009.

¹⁵ To the board belonged, among the others: Noam Chomsky, James Coleman, Robert Dahl, Lionel McKenzie, Frederick Mosteller, and Patrick Suppes.

¹⁶ Among these works, for instance, the first attempt to map congressional preferences

and perhaps the most advanced, from the mathematical point of view, layer of Positive Political Theory in the Eighties and Nineties (Palfrey 2005). By then, one could safely claim that Riker had ultimately concluded his battle to establish American Political Science on formal grounds.

5.2 Some developments of Positive Political Theory

In a review on the “future of a science of politics” (Riker 1977), Riker outlined three ideas that, in his view, were amenable to analytical treatment in a way closely modeled on economic theory. These were: the somewhat intuitive claim that simple-majority elections in single-member districts favor two-party systems; the Median Voter Theorem, and, more broadly, the spatial analysis of voting; and, finally, the “size principle” and coalitional politics. In the previous section, it has been showed what the development of spatial analysis owed to Davis and Hinich (but also Ordeshook and McKelvey) in providing a precise characterization of utility functions for political actors and Euclidean preference space. With respect to the first point—simple-majority elections, also known as “Duverger’s law” (named after the French political scientist Maurice Duverger), the topic was examined in a highly formal fashion from the Eighties onward, using non-cooperative Game Theory (see especially Cox 1997). The case of the size principle and coalitional politics is slightly different. As seen, it was Riker’s most ambitious theoretical contribution. At the same time, it was criticized on both theoretical and empirical grounds, prompting attempts by some of Riker’s associates, such as Shepsle and McKelvey, to reformulate it more precisely, still within cooperative Game Theory but not necessarily through the von Neumann and Morgenstern solution. Coalitional politics still occupies a pivotal role in the formal analysis of politics. Therefore, by briefly addressing one of the cornerstone results in this field, namely David P. Baron and John Ferejohn’s 1989 contribution on bargaining in legislatures (Baron and Ferejohn 1989), the following pages try to identify some lines of development that moved away from Riker’s original result.

5.2.1 Elaborating upon the Size-Principle

The size principle was the main accomplishment of Riker’s analysis of political coalitions. As seen, it states that in n -person zero-sum games, where side-payments are permitted, only minimum winning coalitions occur. This principle means, first, that the strength of a coalition is not a monotonic function of its size; and, second, that in situations where coalitions exceed the minimum winning size, it is rational to expel some members in order to attain the maximum payoff. It was allegedly easy to assess Minimum Winning Coalitions empirically by observing real-world politics or political history. This fulfilled Riker’s aspiration for a “positive theory” of political

in a spatial setting by Rosenthal and Keith Poole (Poole and Rosenthal 1985). Further, all McKelvey’s main papers on the pervasiveness of intransitivities in Euclidean preferences space were an outcome of his years at Carnegie Mellon between 1974 and 1978.

behavior rather than an exclusively mathematical analysis. The importance of this principle in Riker's theoretical work was further reinforced by the apparent weakness of the second part of his analysis, namely the dynamic model of coalition-building. However, the result was also criticized on both theoretical and empirical grounds.

On the "theoretical side," most critiques focused on Riker's treatment of "equilibrium," which he interpreted in terms of the "realizability" of a Minimum Winning Coalition, as well as on the role of side payments and, finally, on the role of transferable utility. Of these critiques, the first two broadened the perspective of Riker's analysis, producing a shift in the toolbox of game-theoretical coalition analysis from the "stable set" solution concept to the "Bargaining Set." Game theorists Robert Aumann and Michael Maschler first introduced this idea in order to integrate existing cooperative solutions (the "stable set" and the "core") with behavioral considerations (Aumann and Maschler 1964). The intuitive idea behind this notion is that the solution of a game is given by a pair consisting of a stable payoff vector and a coalition structure. For a payoff to be "stable," the members of each coalition must be able to defend their payoff against possible objections by their partners.

Political scientist Robert Butterworth attacked the Size Principle, arguing that a more precise perspective should focus on the efforts made by members of a non-minimum winning coalition to avoid being expelled (Butterworth 1971). Although flawed from a game-theoretical standpoint, this intuition led Kenneth Shepsle to investigate the plausibility of the Size Principle through a more mathematically grounded analysis (Shepsle 1974). He therefore introduced a simplified version of the "Bargaining Set." Yet his analysis too was incomplete. Finally, McKelvey and Richard D. Smith were able to provide a consistent mathematical demonstration of the principle by using the "bargaining set" (McKelvey and Smith 1975).

However, in this way, it lost any "positive feature," becoming a purely mathematical result.¹⁷ To overcome this difficulty, McKelvey addressed the problem by dropping the transferable-utility assumption, elaborating, and experimentally testing, together with Ordeshook and Mark D. Winer, a new solution concept for *n*-person games, the "Competitive Solution" (McKelvey, Ordeshook, and Winer 1978).

In a paper published in the *American Political Science Review*, Butterworth disputed Riker's Size Principle by showing that, if side-payments are permitted, then: "*no incentive can be found to cause rational players to reject the initial 'unrealizable' coalition, in favor of an immediately available alternative.*" (Butterworth 1971, p. 741, italics in the text) Significantly, this reframed the function of side payments. In Riker's analysis, a leader uses side payments to add members to a coalition. Butterworth instead considered the possibility of exchange or "bribery" among players. Sup-

¹⁷ This aspect, together with the apparent simplicity of the mathematical result, one may speculate, also explains why this proof was never published but remained in a mimeographed form as a Carnegie Mellon working paper. Nevertheless, McKelvey, in other words, referenced it.

pose a coalition member's loss from being expelled is greater than what it would cost him to remain in the coalition, even with a lower—or even negative—payoff. In that case, the member could “bribe” his companions in order not to be expelled. This point becomes clear through a simple numerical example.¹⁸

Take a five-player zero-sum game.¹⁹ There are three possible partitions (not the total number of winning coalition) of winning coalitions: $([1, 2, 3, 4, 5])$, $([1, 2, 3, 4]; [5])$ and $([1, 2, 3]; [4, 5])$. According to Riker's Size Principle, the expected winning coalition must be the minimum winning one, namely a coalition composed of three members. Assume the following characteristic functions for each partition:

1. $v(1, 2, 3, 4, 5) = 0$
2. $v(1, 2, 3, 4) = 20$ & $v(5) = -20$
3. $v(1, 2, 3) = 30$ & $v(4, 5) = -30$

These yield the following imputations:

1. $v(1, 2, 3, 4, 5) = (0, 0, 0, 0, 0)$
2. $v(1, 2, 3, 4) = (5, 5, 5, 5)$ & $v(5) = (-20)$
3. $v(1, 2, 3) = (10, 10, 10)$ & $v(4, 5) = (-15, -15)$

When side payments are permitted, one can ask if it is possible to identify an incentive for three players within a four-player coalition to expel the fourth and reach a minimum winning coalition. From the values above, a three-player coalition yields a total value of 30, meaning that each player could gain an additional 5. For the fourth player, however, the change is a total loss of 20, since he goes from receiving 5 as a member of the winning coalition to -15 as a member of a losing coalition. He may therefore prefer to avert this loss by offering, for example, 6 to each of the other coalition members in order to stay. The four-player coalition is still valued at 20, but the imputation becomes: $(11, 11, 11, -13)$. This clearly dominates the imputation $(10, 10, 10, -15)$, which corresponds to what each of the three players would receive in the winning coalition $v(1, 2, 3)$ and what the fourth player would lose as part of the losing coalition $v(4, 5)$. Therefore, the winning coalition's size will be 4 rather than 3, contrary to the Size Principle.

As Butterworth summarized: “the payoffs as given by the rules to the three-person winning coalition dominate those given to the four-person winning coalition, as Riker says; *but* the payoffs of the four-person winning

¹⁸ Note that Shepsle, as well as McKelvey and Smith, adopted the same numerical example in building upon Butterworth's idea.

¹⁹ Butterworth's analysis is not axiomatic. Other than zero-sum, this game needs to be symmetric. McKelvey also analyzed the simple games. A simple game is a game where only two possible outcomes are possible: winning and losing. Formally, $v(B) = 0$ or $1 \quad \forall B \in P(N)$ ($P(N)$ is the set of all subsets of N)

coalition with bribing in operation dominate those of the three-person coalition without bribing.” (Butterworth 1971, p. 742) He also showed that this conclusion holds for at least two of the three situations identified by Riker: when the value of the winning coalition decreases as its size grows (the example above: in Riker’s terms, when the winning coalition’s characteristic function is negatively sloped) and when the slope is zero.

Notice that Butterworth did not reject Riker’s analysis completely, but rather generalized it by proposing a new principle, the “maximum number of positive gainers principle”. This principle says nothing about the size of winning coalitions. Instead, it asserts that there is a range within which the number of positive gainers must fall (and that positive gainers must be members of minimum winning coalitions). In other words, even if there is a positive incentive, say b , for a coalition to expel members in order to reach the minimum winning size, it is not necessary to expel members in order to gain b . Butterworth claimed that each example explained by the “Size Principle” is equally explained by his principle, which, in addition, also clarifies why some winning coalitions may assign negative payoffs to their members.

Riker replied in the same issue of the *American Political Science Review*, focusing on an apparent inconsistency in Butterworth’s argument. To claim that a four-player coalition can yield payoffs (11, 11, 11, -13) is equivalent to claiming that a three-player coalition worth 33 exists, which violates the rules of the game, since a three-person coalition is worth 30 (Riker 1971).

Nevertheless, Butterworth had raised several important theoretical issues that needed to be incorporated into a more mathematically rigorous analysis of the principle. This was the aim of Shepsle’s paper, published in 1974, in the same *Journal* of the previous contributions.

Shepsle noted that both Riker’s original argument and Butterworth’s critique suffered from two major problems. First, the inherent instability of zero-sum essential games; second, the way both authors had used game theory. As he wrote:

"To speak of equilibrium in a game $G(n, v)$ is to have some solution in mind, the elements of which possess certain properties of stability. Having specified a solution concept, and having assured ourselves that the game $G(n, v)$ possesses a solution, it is then (and probably only then) appropriate to ask whether anything general may be said about the size of winning coalitions. Neither Riker nor Butterworth tie their analysis explicitly to a formal solution to $G(n, v)$ and this, I believe, has been a source of some confusion." (Shepsle 1974, p. 508, italics in the text.)

Neither Riker’s nor Butterworth’s reasoning was grounded in a formal solution concept. Riker’s reliance on the stable set (von Neumann) and Morgenstern’s original solution concept for n -person games) was justified mainly by his dissatisfaction with many subsequent solution concepts (beginning with the “core” and the “Shapley Value”) (Riker 1962b, pp. 38–9). Yet this choice weakened the general mathematical consistency of his result. Shepsle’s inquiry therefore addressed whether a coalition size could be

identified that, by itself, sustained an imputation in the stable set. Using Butterworth's numerical example, the stable set of the game is:

$$V = \{(10, 10, 10, x, -30 - x) \mid -20 \leq x \leq -10\}$$

This set is infinite, because the losses are determined only up to an interval. Yet there is no explicit mention of the coalition structure that generates particular elements as outcomes of the game. Any coalition whose size is greater than or equal to the Minimum Winning Coalition can sustain an imputation in V .²⁰ Since coalitions of three, four, or five members can sustain imputations in the stable set, coalition size cannot be determined solely on this basis. This result exposes the flaw in Riker's analysis.

To address this problem, Shepsle first introduced two behavioral constraints. First, he defined the *acceptable coalition structure* for each outcome in V .²¹ Second, each player is constrained only by the characteristic function. In doing so, Shepsle accepted one of Butterworth's central points, namely the possibility of bribery by coalition members. He argued that, given a subset V^* of V and the set:

$$V - V^* = \{(10, 10, 10, x, -30 - x) \mid -20 < x < -10\}$$

any coalition C with $|C| = 4$ fails to satisfy the first constraint. Therefore, if the outcome of the game lies in $V - V^*$, only coalition structures that include a Minimum Winning Coalition are acceptable. But this is not the case for $V^* = \{(10, 10, 10, -10, -20)\}$ (Shepsle 1974, p. 513). Hence: "the size principle will be strongly sustained in this game if there is some reason to exclude the $y \in V^*$ as likely outcomes. This can be done through a careful specification of the second constraint such that the outcome of the game is among the elements of a subset of $V - V^*$. Hence, the only acceptable coalition structures are those that contain a Minimum Winning Coalition." (ibidem) In other words, the Size Principle depends heavily on "sociological" considerations about the structure of the game.

To reinforce his argument, Shepsle adopted an idea closely related to another cooperative solution concept, namely the "Bargaining Set." Unlike the Stable Set, this concept specifies the coalition structure that sustains a given payoff vector. That is, given a game $G(n, v)$, an outcome is a *payoff configuration* $(\vec{x}; \vec{K})$ consisting of a payoff vector and a coalition structure:

1. If $x_i \geq v(i)$ for all x in $(\vec{x}; K)$, then it is an *individually rational payoff configuration* (IRPC)
2. If $\sum_{i \in K_j} x_i \geq v(K_j)$, for all K_j in $(\vec{x}; K)$, then it is an *acceptable payoff configuration* (APC)

²⁰ To better see this point, take $|C| = 3$ (which is a Minimum Winning Coalition). Then $10 + 10 + 10 \leq 30 = v(1, 2, 3)$. Take instead $|C| = 4$. Then $10 + 10 + 10 + x \geq 20 = v(1, 2, 3, 4)$ is not part of V . Therefore, $|C| = 4$ is sustained by V .

²¹ Formally, it means that each coalition that is formed must receive at least their values as specified by the characteristic function: $\sum_{i \in K_j} x_i > v(K_j)$, where $K = (K_1, K_2, \dots, K_m)$ are partitions of the set of players.

Any element in the stable set, together with a coalition structure K_1 that contains only the three positive gainers, is both an IRPC and an APC.²²

Since, in Shepsle's view, Butterworth's most valuable point was precisely the internal activity of coalition members (and Shepsle's second behavioral constraint addressed this issue), the bargaining-set framework was well suited to operationalize it.

Formally, consider a IRPC $(x; K)$, and $s, t \in I$. Player s has an *objection* against t (denoted $[y^s(t); B]$) if there exists a coalition B , with $s \in B$ and $t \notin B$, and a payoff vector y satisfying the following three conditions:

$$\sum_{i \in B} y_i = v(B)$$

$$y_s > x_s$$

$$y_i \geq x_i \quad \forall i \in B$$

Player t has a *counterobjection* to $[y^s(t); B]$ if there exists a coalition C , with $t \in C$ and $s \notin C$, and a distribution z such that:

$$\sum_{i \in C} z_i = v(C)$$

$$z_i \geq x_i \quad \forall i \in C$$

$$z_i \geq y_i \quad \forall i \in B \cup C$$

If every objection to an IRPC $(x; K)$ has a counterobjection, then $(x; K)$ is *efficacious*. Shepsle finally showed that no vector in V^* is efficacious and that the only IRPCs in V that are both APC and efficacious are those in which "*both gains and losses are symmetrically shared by a minimum winning coalition and a maximum losing coalition.*" (Shepsle 1974, p. 515)

Although he restricted his analysis to the five-player game and did not generalize it, he concluded:

"One message, in any event, is clear. It is unlikely that much can be said with confidence about coalition structure without analytically viable (and perhaps sociologically rich) solution concepts. From Butterworth's initial research and the analysis presented in the first three sections of this paper, it appears that minimum winning coalitions constitute *unstable* equilibrium points in n -person zero-sum games. That is, there appear to be forces in the coalition formation process that drive winning coalitions toward minimal size, but fail to keep

²² To see this point: IRPC is satisfied when: $x_1 = 10 > -20$; $x_2 = 10 > -20$; $x_3 = 10 > -20$; APC is satisfied when: $10 + 10 + 10 \geq 30$ and $x - 30 - x = -30$

them at that point. If, however, the usual assumptions about n -person zero-sum coalition processes are supplemented with assumptions about coalition intentions and capabilities, there are good reasons to expect minimum winning coalitions in all but the most extreme instances." (Shepsle 1974, p. 515, italics in the text)

In their unpublished "Comment on The Debate over Riker's 'Size Principle,'" McKelvey and Smith objected to Shepsle's analysis on two technical points. First, he had treated the bargaining set as a subset of the stable set, which is valid only under certain conditions. Further, in his example, objectors and counterobjectors did not belong to the same coalitions, contrary to what the theory requires (McKelvey and Smith 1975). However, as they wrote: "Despite the above difficulties, we feel that Shepsle is basically on the right track, and his argument that the size principle and the MPG [maximum number of positive gainers] principle are not based on any formal solution concept is correct." In particular, they showed that in a large class of games the size principle follows directly from the assumption of "internal stability" of coalitions, which bargaining-set theories implement through the requirement of "coalitionally rational payoff configurations." (McKelvey and Smith 1975, pp. 3-4)

Formally, a payoff configuration $(\mathbf{x}, B) = (x_1, \dots, x_n; B_1, \dots, B_m)$ is a pair with $x \in \mathbb{R}^n$ and B a partition of N such that $\sum_{i \in B_k} x_i = v(B)$ for $k = 1, \dots, m$. Here, x_i represents the payoff to player i . That is, coalitions that form must receive their value. For internal stability of payoff configurations, two stronger conditions are required:

1. Individual rationality if and only if $x_i \geq v(i) \quad \forall i \in N$. This condition requires that every individual receive his value.
2. Coalitional rationality if and only if $\sum_{i \in T} x_i \geq v(T) \quad \text{for } T \subseteq B_k \in B$. This condition requires that every subset of an extant coalition receive its value.²³

McKelvey and Smith showed that the internal-stability conditions implied by coalitional rationality generate the size principle. The result can be understood by returning to the standard numerical example introduced by Butterworth. In that example, only three-person coalitions within the coalition structure receive their value (i.e., are coalitionally rational payoff configurations). Indeed, any three-member subset of a larger four-person coalition receives less than the value it could obtain by forming a coalition on its own (since within a four-person coalition any three members jointly receive 15, whereas a three-person coalition receives 30). Hence, a winning coalition maximizes its value by reducing its size to the minimum winning size. Moreover, the same logic applies to four-member winning coalitions (namely, those in which the fourth player "bribes" his way into the coalition).

²³ Note the slight change from Shepsle's. His analysis rested upon individual rationality and the "acceptability" of some payoff configurations. The second condition refers to a partition of I . Instead, McKelvey refers to a subset of a coalition in a partition.

While the result held for the example above, the two authors generalized it to a broad class of games, especially simple and symmetric games.²⁴ They added two further formal conditions. The first concerns how a coalition's value changes when an additional player joins. Suppose a player is added to a minimum winning coalition and the marginal value added is less than the proportional share each player is receiving. In that case, the game exhibits "decreasing returns to scale."²⁵ The second condition concerns the "inessentiality" of a subset of players within a coalition. A set of players is inessential in a coalition if they can do just as well by not joining. They then proved two main theorems: in a simple game, if a payoff configuration is coalitionally rational for some $B_k \in B$ with $B_k \in W$, then either $B_k \in M$, or some individuals in B_k are inessential for the coalition (that is, a winning coalition is either minimal, or some individuals can obtain the same payoff by not joining it). The second theorem concerns symmetric games with decreasing returns to scale (again under coalitionally rational payoff configurations) and states that, in this case, if a coalition is winning, then it is also minimal winning.

These results illustrate how fruitful Riker's insights were. Moreover, in the late Sixties and early Seventies, other young scholars, such as the future Nobelist Robert Wilson, then a graduate student at the Stanford Business School, began to analyze social choice theory and voting through cooperative game theory (Wilson 1971; Wilson 1972).²⁶ However, it also became apparent that, once fully axiomatized, the size principle and coalition theory lost much of their simplicity and their alleged predictive power.

5.2.2 The game-theory revolution and its impact on "Positive Political Theory."

Coalition theory occupied a central role in formal political theory. The attempts to solve the difficulties discussed above, then, followed two main paths. The first remained within cooperative Game Theory but abandoned the crucial assumption of transferable utility. The second adopted bargaining theory in extensive-form games, namely, embraced the most recent developments in non-cooperative Game Theory. The first line of research includes the new solution concept for n -person games without transferable utility, namely McKelvey's and Ordeshook's "Competitive Solution" (McKelvey, Ordeshook, and Winer 1978; McKelvey and Ordeshook 1978). The second path is exemplified by one of the most influential models in

²⁴ Symmetric games are games where the value of a characteristic function depends only on the number of players. Finally, note that a simple game cannot be zero-sum (because if $v(B) = 1$, then $v(B^C) = 0$, whereas for the zero-sum condition, it should be -1).

²⁵ Formally: if, for all $T \subseteq S \subseteq N$ and $T \in M$, then $\frac{v(S)}{|S|} > \frac{v(T)}{|T|}$ (M is the set of all minimum winning coalitions). If the condition above holds for all $T \in W$ (the set of all winning coalitions), McKelvey spoke of "uniformly decreasing return to scale."

²⁶ That the issues of coalitions and voting are tied can be evident by looking at a majority of voters as a "winning coalition" and the outcome of the voting process as an imputation. For instance, the simplest example relates the lack of a Condorcet Winner (the "Condorcet Paradox") to the emptiness of the core.

formal political science: David P. Baron's and John Ferejohn's analysis of bargaining in legislatures (Baron and Ferejohn 1989).

A passage from Ordeshook is worth quoting in full:

"Briefly, bargaining and cooperation are studied formally using one of two approaches. The first (classical) approach is the one illustrated by the definitions of the V -set, Bargaining Set, and Competitive solution, whereby more or less ad hoc restrictions are used to define a subset of the feasible set of imputations or proposals, possibly with some attempt at a behavioral justification. The idea here, of course, is to abandon the idea of point predictions and, by focusing on the properties of sets of outcomes rather than on the specific elements of those sets taken one at a time, to narrow the range of likely outcomes. The second approach accepts the critique of the classical approach that a set-theoretic formulation fails to address the issue of how agreements are enforced (if cooperative agreements are enforceable, the argument goes, it must be the case that they are part of an equilibrium to some appropriately conceptualized non-cooperative game), and proceeds instead with an explicit model of the bargaining process in extensive form." (Ordeshook 2007, p. 183)

In cooperative games, the assumption of transferable utility is contained directly in the definition of the characteristic function. This is, as seen, a real-valued function that assigns to each coalition a numerical value. This value represents the quantity of utility that coalition members can secure and distribute among themselves. If utility is assumed to be linear in money and side payments are allowed, then the coalition's value is linear in the exchanges among players.²⁷ McKelvey and others showed that this assumption becomes untenable when preferences are Euclidean. In such cases, it can be shown that for some groups of voters the value of their coalition cannot be summarized by a single number (McKelvey, Ordeshook, and Winer 1978, pp. 194–5). This observation calls for redefining the characteristic function as a set-valued function rather than a real-valued one, that is, $v(S) \subseteq \mathbb{R}^n$.

Once the assumption of transferable utility is abandoned, however, many results in cooperative coalition theory no longer hold. To address this issue, McKelvey and Ordeshook developed a new solution concept for n -person games without transferable utility, namely the "Competitive Solution." They explored this idea analytically in a paper presented at a conference on Game Theory and Political Science sponsored by the Mathematical Social Science Board of the Social Science Research Council (McKelvey and Ordeshook 1978). In addition, they published a paper with a third political scientist, Mark Winer, in which the concept was also tested experimentally (McKelvey, Ordeshook, and Winer 1978).

²⁷ Note, however, that transferable utility is a technical hypothesis and does not preclude that players exchange resources among themselves.

The rationale behind this new solution was the idea that “potential coalitions must bid for their members in a competitive environment via the proposals they offer.” Since several coalitions attempt to form simultaneously, an efficient bid rewards the pivotal members of each coalition (McKelvey, Ordeshook, and Winer 1978, p. 200). A proposal by a coalition $C \subseteq N$ is defined as an ordered pair (u, C) such that $u \in v(C)$ and $u \in v(N)$. A proposal (u_1, C_1) is considered viable if, for any other proposal (u_2, C_2) , it is not the case that $u_1 <_i u_2$ for all $i \in C_1 \cap C_2$, that is, for the set of pivotal players common to both coalitions. In other words, a proposal is viable if the pivotal players do not strictly prefer another proposal over the one in which they are pivotal. Let K be a set of proposals. A proposal is viable in K if it is viable against all proposals in K .

To define the “Competitive Solution,” two additional conditions are required. The first concerns the set K . If each coalition in K has exactly one proposal, and if all proposals in K are viable against one another, then K is said to be balanced. Since multiple balanced sets may exist, the proponents of the competitive solution focused on those balanced sets that are not upset by any alternative proposal—that is, proposals that are “as attractive as possible” to their pivotal members. A “Competitive Solution” is therefore defined as a set K that is balanced and not upset by any proposal (u, C) .

Beyond its formal properties, the competitive solution was also intended to improve the predictive power of coalition theory. As McKelvey observed, “concepts such as the V -set are principally mathematical abstractions without behavioral rationale. Hence, it is difficult to assess their applicability when particular rules constrain bargaining or negotiation procedures in committees.” (McKelvey, Ordeshook, and Winer 1978, p. 189) The main advantage of the competitive solution, therefore, lay in its potential for empirical testing.

These claims may appear surprising given McKelvey’s strong commitment to highly formal analysis. However, they closely echo Riker’s emphasis on positive political analysis and on the need for analytically precise and empirically grounded theories. Another point of continuity with Riker’s approach was the continued reliance on cooperative game theory. Yet, as Riker himself later remarked, “unfortunately, this enterprise comes just at the time that, in economics, theorists are abandoning cooperative theory for non-cooperative theory.” (Riker 1992, p. 219) He was referring, of course, to the game-theoretic revolution that reshaped economics from the late Seventies and early Eighties onward.

One of the forces driving that revolution was precisely the perceived lack of strong behavioral foundations in cooperative Game Theory. An issue only partially addressed by solution concepts such as the bargaining set.²⁸

Perhaps nothing illustrates how marginal non-cooperative Game Theory remained for political scientists as late as the Seventies better than a look

²⁸ Another factor was the development of the Nash equilibrium concept in the Sixties and Seventies, including its extension to extensive-form games.

at Riker's and Ordeshook's textbook *An Introduction to Positive Political Theory* (Riker and Ordeshook 1973).²⁹ Of the twelve chapters in the book, four are devoted to game theory, amounting to 124 pages. More striking, however, is the way the material is organized. One chapter deals with n -person games, one with power indices, one with the size principle, and one with two-person games. The classification does not revolve around the key distinction between cooperative and non-cooperative games, that is, whether communication is unrestricted and binding agreements are possible, but rather around the number of players. Under the heading of n -person games, the authors discuss only coalitional games, without even mentioning non-cooperative n -person games. Non-cooperative analysis and the Nash equilibrium appear only in the context of two-player games, alongside the bargaining problem (a cooperative solution). In the authors' taxonomy, two-person games are divided into zero-sum and non-zero-sum, finite and infinite, cooperative and non-cooperative. Yet very little attention is devoted to the Nash Equilibrium itself, partly because in the types of games emphasized, namely zero-sum, finite, non-cooperative games, the Nash Equilibrium coincides with von Neumann's minimax solution. Notably, no attention at all is given to the cutting-edge work of Harsanyi and Selten on extensive-form games and incomplete information.

Neglecting Harsanyi's and Selten's contributions greatly diminishes the relevance of game theory for theoretical analysis in both economics and political science, as David Kreps's words from 1991 clearly indicate:

"The recent impact of game-theoretic methods in economics [...] traces in large measure from the ability to think about the dynamic character of competitive interactions by using extensive form games. [...] *The great successes of game theory in economics have arisen in considerable measure because game theory gives us a language for modeling and techniques for analyzing specific dynamic competitive interactions.*" (Kreps 1991, pp. 50–1, italics in the text)

Why, then, did Riker and Ordeshook overlook these developments? Several explanations are possible. One is that these contributions were not easily accessible to non-specialists. Selten's early work, for instance, was written in German and remained untranslated, even though he later expanded his ideas in a major 1975 paper published in the *International Journal of Game Theory*. Harsanyi, although part of the American economics community and a Ph.D. student of Kenneth Arrow at Stanford, was based at Berkeley's Business School and thus somewhat peripheral to the core of economic theory. Nonetheless, the community of game theorists was small, and both Selten and Harsanyi maintained close ties with figures such as Aumann, Maschler, and Shapley. Another explanation concerns Riker's and Ordeshook's primary sources for game theory. Apart from the canonical but increasingly outdated work of Luce and Raiffa, their

²⁹ A similar argument can be raised for another comprehensive review of game theory and politics, that of Steven J. Brams (Brams 1975)

main references were the introductory and non-technical texts by Anatol Rapoport and Morton Davis, published in 1966 and 1970.

These explanations, however, are not fully satisfactory. As shown earlier, Harsanyi had some contact with the public choice community, and Riker had acquaintances among game theorists in the late Sixties. More importantly, while Riker's command of advanced mathematics was limited, Ordeshook's was not. His knowledge of game theory exceeded what the joint textbook suggests. Indeed, although game theory did not occupy a central place in the Rochester economics department, a common feature of most economics departments in the Sixties and Seventies, the department did include the young economist and game theorist James Friedman. While less famous than Harsanyi or Selten, Friedman is well known for his critical applications of game theory to market theory.³⁰ Friedman helped Ordeshook to expand his understanding of game theory well beyond the introductory level of Luce and Raiffa (Ordeshook 2021). Thus, when Ordeshook worked on the textbook, his knowledge of game theory was probably unmatched among political scientists, with the exception of McKelvey.

The most convincing explanation for Riker's and Ordeshook's relative indifference to non-cooperative Game Theory lies perhaps elsewhere. Scientific innovations are rarely adopted immediately, and Harsanyi's and Selten's contributions did not settle the problem of how to model non-cooperative extensive-form games.³¹ It is therefore not surprising that Riker remained attached to the form of Game Theory that had originally inspired him, namely coalitional games. Politics, in Riker's view, involves both enforcement and the aggregation of interests.³² One of the most appealing features of non-cooperative Game Theory is that Nash equilibria are self-enforcing, a feature closely resembling the functioning of competitive markets. In politics, however, choices and actions are enforced by institutions, and institutions themselves are the product of deliberate and binding agreements. Despite its reliance on somewhat loose behavioral assumptions, *n*-person cooperative theory allows one to focus on the properties of institutional arrangements and provides a natural framework for combining heterogeneous interests through imputations and set-valued solutions.

Subsequent developments in non-cooperative Game Theory showed that similar insights could be achieved using point-valued solutions such as Nash Equilibrium. Not surprisingly, the most successful results also incorporated cooperative elements, as in bargaining theory. The development of such models was one of the major achievements of game theory in the Eighties. One of the most influential of these models is Baron's and Ferejohn's analysis

³⁰ He also contributed to the first formal proof of the so-called "Folk Theorem," a foundational class of results relating subgame-perfect Nash equilibria to the length of games.

³¹ Consider, for example, the notion of Subgame Perfect Equilibrium, Perfect Bayesian Equilibrium and Sequential Equilibrium, developed in the Eighties.

³² In the introduction to their textbook, the authors identify three core political processes: "the selection of society's preferences, the enforcement of the choices that revealed them, and finally, the production of goals or outputs that embody the choices."(Riker and Ordeshook 1973, pp. 2-7)

of legislative bargaining.

They published their work in the *American Political Science Review* in 1989 (Baron and Ferejohn 1989). The central issue they addressed was endogenous agenda formation in a unicameral legislature operating under majority rule, which they modeled using non-cooperative Game Theory. Social choice theory has shown that voting equilibria are often unattainable in multidimensional settings, making it possible, following McKelvey's most famous result, for an agenda setter to engineer virtually any outcome by sequencing votes appropriately. Earlier, in 1979, Shepsle had shown how equilibrium could be achieved in multidimensional spaces by structuring decisions across multiple issues, thereby guaranteeing a Condorcet winner for each single-issue vote, an idea he termed "structure-induced equilibrium" (Shepsle 1979). Shepsle thus recast a social choice problem as an institutional one, focusing on committee systems in legislatures. His equilibrium, however, remained "based on a concept of stability drawn from the field of social choice." (Baron and Ferejohn 1989, p. 1181)

Like Shepsle, Baron and Ferejohn adopted an institution-based approach, emphasizing the sequential processes of proposal making, amendment, and voting. Unlike him, however, they modeled these processes as a non-cooperative, multi-session game and demonstrated the existence of a Nash Equilibrium. Moreover, the equilibrium outcome varies with institutional rules, particularly whether the legislature operates under a "closed rule" or an "open rule." Under a closed rule, amendments are not allowed, and proposals are voted directly against the status quo, yielding only win-or-lose outcomes. Under an open rule, amendments may be proposed, and the identity of the member who proposes or calls for a vote, the "recognition rule", becomes crucial. At the beginning of a session, each member i has a probability p_i of being recognized. Given a feasible set of proposals X , a proposal is a vector $x^i = (x_1^i, \dots, x_n^i)$ such that $\sum_{j=1}^n x_j^i$. In a closed-rule system, only one member is recognized; in an open-rule system, another member j may be recognized with probability $\frac{p_j}{\sum_{k \neq i} p_k}$ and propose an amendment. The resulting model is therefore substantially more complex.

Baron and Ferejohn represent each legislator's preferences using the utility function $u^j(x^k, t) = \delta^t x_j^k$, where t denotes the session in which the allocation x^k is adopted. Each player's pure strategy s_τ^i is a set of functions mapping the history H_τ (the sequence of events up to time τ) into the set of proposals X if player i is recognized, or into the set {yes, no} otherwise. Mixed strategies are probability distributions over pure strategies. Each player also has a continuation value $\delta v_i(t, g)$, representing the value to player i if the legislature moves to subgame g .

The model describes a bargaining process in which proposals are made and either accepted or rejected. As discussed earlier, Nash provided the first systematic analysis of bilateral bargaining (Nash 2002b). Rubinstein later developed an axiomatic model of bilateral bargaining, in which two players bargain over splitting a dollar (Rubinstein 1982). In Rubinstein's model, the first player proposes a division $[p, 1 - p]$. If accepted, the game ends; otherwise, the second player makes a counterproposal, and

the process continues. Each player is impatient, with a discount factor δ reducing payoffs over time. The game is non-cooperative and features complete information, with strategies defined as sequences of proposal and response functions. Because the game is multistage, equilibrium must be subgame perfect. Rubinstein showed that the unique equilibrium outcome involves immediate acceptance of the first proposal. This result generated an extensive literature.

Baron and Ferejohn's model belongs to this tradition but differs in important respects. First, Rubinstein's model is bilateral, whereas Baron and Ferejohn's is multilateral. Second, impatience plays a different role: in Rubinstein's model, equilibrium allocations converge to equality as impatience diminishes, whereas no analogous result emerges in Baron and Ferejohn's framework. Third, equilibrium outcomes depend explicitly on institutional rules. Under a closed rule and a finite horizon, the first proposal is accepted, resembling Rubinstein's result. Under an open rule, however, outcomes depend on factors such as impatience and the number of legislators.

One of the most influential findings of Baron and Ferejohn's model is that amendment rules affect the equity of outcomes and the distribution of resources. As they state:

"With a closed rule, the equilibrium outcomes are majoritarian, the first proposal is passed, and benefits are distributed to a minimal majority. Since each member acts noncooperatively, the majority is not a coalition in the sense the term is used in cooperative game theory. Compared to a closed rule, the opportunity to make an amendment under an open rule reduces the agenda power of the member recognized first and results in an outcome that more evenly distributes the benefits among the majority. Unless impatience is great, however, the distribution is majoritarian, and the proposal made by the first member recognized is not necessarily accepted. If, however, the number of members is small and there is substantial impatience [...], the equilibrium is "universalistic" in the sense that every member receives benefits." (Baron and Ferejohn 1989, pp. 1199–1200)

It is important to stress that this result does not directly contradict Riker's Size Principle.³³ Minimal majorities may still arise. However, the nature of the result is fundamentally different from Riker's. Earlier, we argued that Riker's coalition analysis was flawed in two respects. First, the Size Principle, although intuitively appealing, lacked a solid game-theoretic solution concept and behavioral foundation. Second, Riker's "dynamical model" failed to articulate the underlying bargaining process clearly. Through the bargaining set and the competitive solution, scholars such as Ordeshook and McKelvey addressed the first issue. But, as Baron and Ferejohn observed, "Cooperative models of politics abstract from the

³³ A table in Baron and Ferejohn's paper reports equilibrium outcomes for different discount factors and legislature sizes (Baron and Ferejohn 1989, p. 1197)

process by which alternatives arise and assume that coalitions will freely form to defeat alternatives when a majority of members prefers another available alternative.” (Baron and Ferejohn 1989, p. 1200) Recasting the problem as a non-cooperative bargaining game allowed Baron and Ferejohn to highlight the sequential nature of legislative decision-making and to accomplish what Riker had attempted nearly thirty years earlier. The result, like much else in this story, underscores the ambivalent legacy of Riker’s work: on the one hand, its limitations, perhaps rooted in excessive optimism about cooperative Game Theory; on the other, the profound insights it generated for the analysis of political phenomena in a new, more scientific outlook.